

CACAO seminar – March 2009

Hardware Operators for Pairing-Based Cryptography

— Part II: Because speed also matters —

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CACAO, LORIA, Nancy, France

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CSD, IPN, Mexico City, Mexico

Outline of the talk

- ▶ Previously in the Jean-Luc Beuchat Tour
- ▶ A closer look at the algorithm
- ▶ Accelerating the η_T pairing
- ▶ Accelerating the final exponentiation
- ▶ Implementation results
- ▶ Concluding thoughts

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Bilinear pairings

- ▶ $\mathbb{G}_1 = \langle P \rangle$: additively-written cyclic group of prime order $\#\mathbb{G}_1 = \ell$
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- ▶ A bilinear pairing on $(\mathbb{G}_1, \mathbb{G}_2)$ is a map

$$\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$$

that satisfies the following conditions:

- non-degeneracy: $\hat{e}(P, P) \neq 1_{\mathbb{G}_2}$ (equivalently $\hat{e}(P, P)$ generates \mathbb{G}_2)
- bilinearity:
$$\hat{e}(Q_1 + Q_2, R) = \hat{e}(Q_1, R) \cdot \hat{e}(Q_2, R) \quad \hat{e}(Q, R_1 + R_2) = \hat{e}(Q, R_1) \cdot \hat{e}(Q, R_2)$$
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 - computability: \hat{e} can be efficiently computed
- ▶ Immediate property: for any two integers k_1 and k_2
- $$\hat{e}(k_1 P, k_2 P) = \hat{e}(k_2 P, k_1 P) = \hat{e}(P, P)^{k_1 k_2}$$

Pairings in cryptography

- ▶ At first, used to attack supersingular elliptic curves
 - Menezes-Okamoto-Vanstone attack, 1993
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Pairings in cryptography

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 - Menezes-Okamoto-Vanstone attack, 1993
 - Frey-Rück attack, 1994
- ▶ One-round three-party key agreement (Joux, 2000)
- ▶ Identity-based encryption
 - Boneh-Franklin, 2001
 - Sakai-Kasahara, 2001
- ▶ Short digital signatures
 - Boneh-Lynn-Shacham, 2001
 - Zang-Safavi-Naini-Susilo, 2004
- ▶ ...

The Tate pairing over supersingular elliptic curves

► We first define

- \mathbb{F}_{p^m} , a finite field, with $p = 2$ or 3
- E , a supersingular elliptic curve defined over \mathbb{F}_{p^m}
- ℓ , a large prime factor of $\#E(\mathbb{F}_{p^m})$

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- ▶ $\mathbb{G}_1 = E(\mathbb{F}_{p^m})[\ell]$, the \mathbb{F}_{p^m} -rational ℓ -torsion of E :

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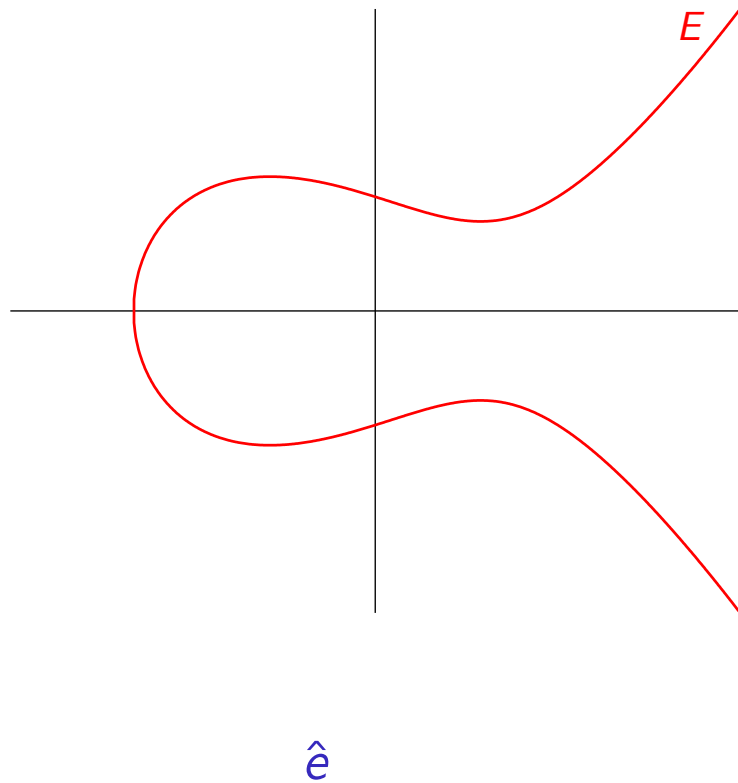
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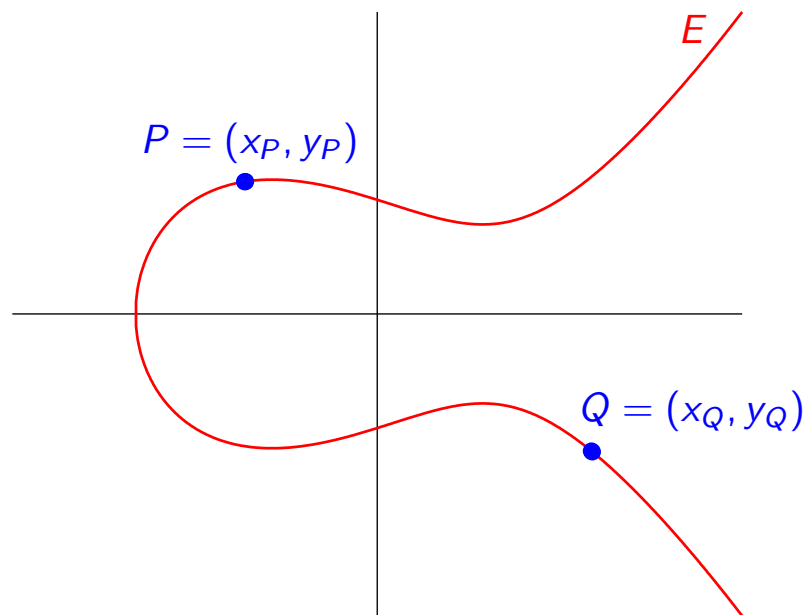
► k is the embedding degree, the smallest integer such that $\mu_\ell \subseteq \mathbb{F}_{p^{km}}^\times$

- $k = 4$ in characteristic $p = 2$
- $k = 6$ in characteristic $p = 3$

The Tate pairing over supersingular elliptic curves

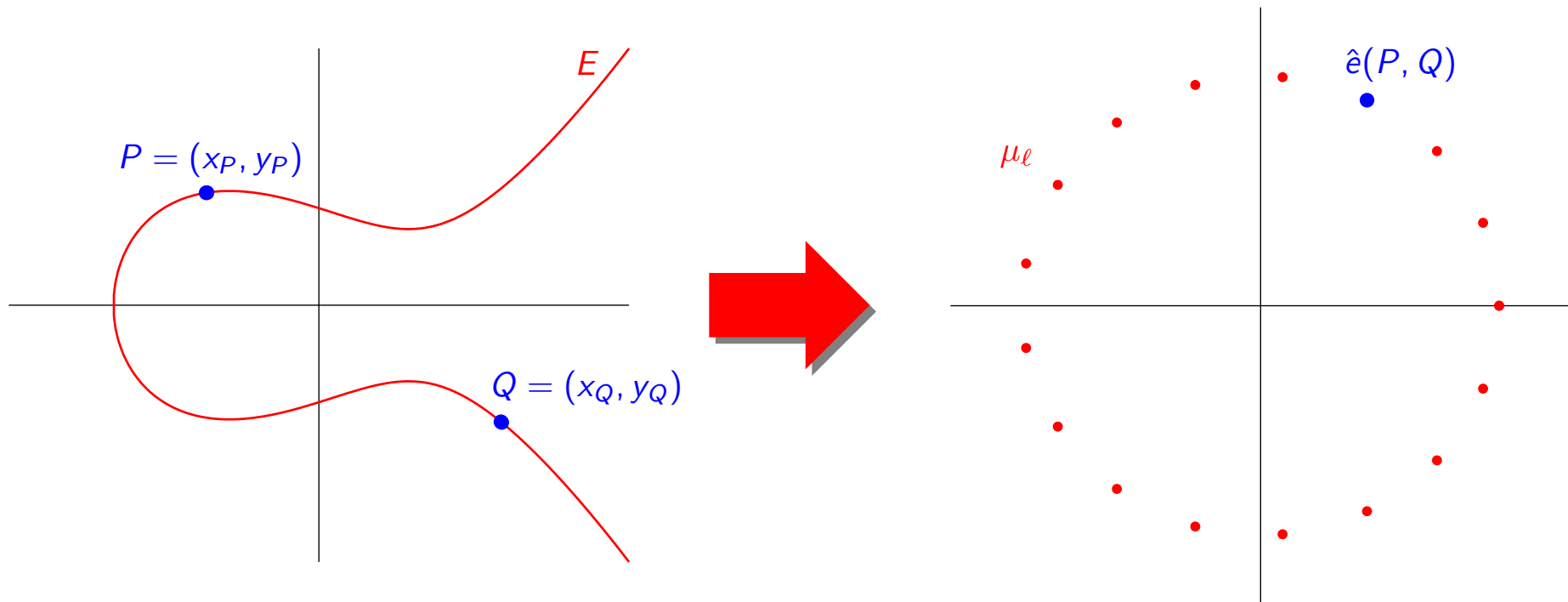


The Tate pairing over supersingular elliptic curves



$$\hat{e} : E(\mathbb{F}_{p^m})[\ell] \times E(\mathbb{F}_{p^m})[\ell] \\ (P , Q)$$

The Tate pairing over supersingular elliptic curves



$$\hat{e} : E(\mathbb{F}_{p^m})[l] \times E(\mathbb{F}_{p^m})[l] \longrightarrow \mu_l \subseteq \mathbb{F}_{p^{km}}^\times$$

$$\left(P, Q \right) \longmapsto \hat{e}(P, Q)$$

Security considerations

$$\hat{e} : E(\mathbb{F}_{p^m})[\ell] \times E(\mathbb{F}_{p^m})[\ell] \rightarrow \mu_\ell \subseteq \mathbb{F}_{p^{km}}^\times$$

- ▶ The discrete logarithm problem should be hard in both \mathbb{G}_1 and \mathbb{G}_2

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Base field (\mathbb{F}_{p^m})	\mathbb{F}_{2^m}	\mathbb{F}_{3^m}
Lower security ($\sim 2^{64}$)	$m = 239$	$m = 97$
Medium security ($\sim 2^{80}$)	$m = 373$	$m = 163$
Higher security ($\sim 2^{128}$)	$m = 1103$	$m = 503$

- ▶ \mathbb{F}_{2^m} : simpler finite field arithmetic
- ▶ \mathbb{F}_{3^m} : smaller field extension

Computation of the Tate pairing

$$\hat{e} : E(\mathbb{F}_{p^m})[\ell] \times E(\mathbb{F}_{p^m})[\ell] \rightarrow \mu_\ell \subseteq \mathbb{F}_{p^{km}}^\times$$

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- polynomial basis: $\mathbb{F}_{p^m} \cong \mathbb{F}_p[x]/(f(x))$
- $f(x)$, degree- m polynomial irreducible over \mathbb{F}_p

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- ▶ Arithmetic over $\mathbb{F}_{p^{km}}^\times$:
 - tower-field representation
 - only arithmetic over the underlying field \mathbb{F}_{p^m}

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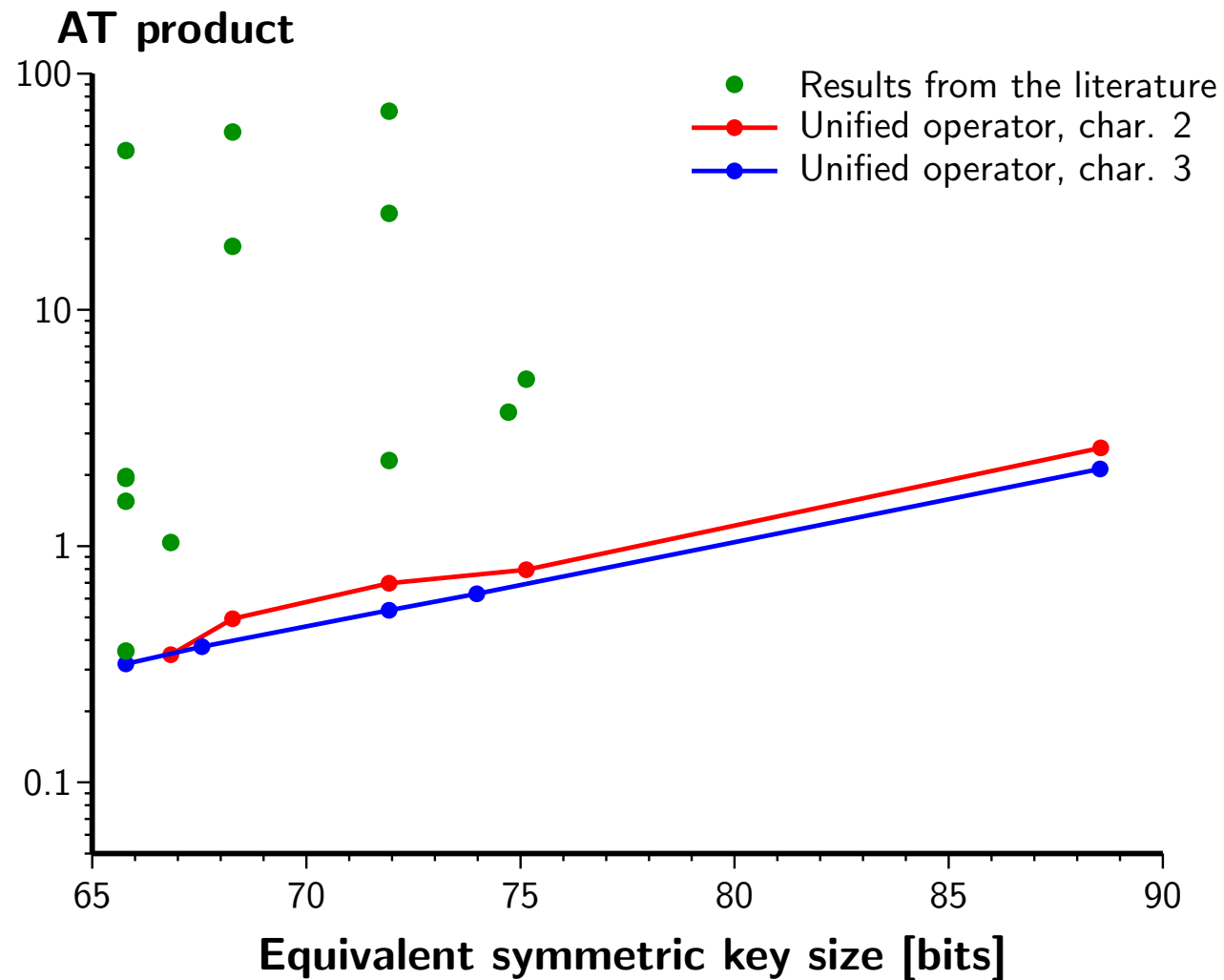
- ▶ Operations over \mathbb{F}_{p^m} :
 - $O(m)$ additions / subtractions
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 - $O(m)$ Frobenius maps ($a \mapsto a^p$, i.e. squarings or cubings)
 - 1 inversion

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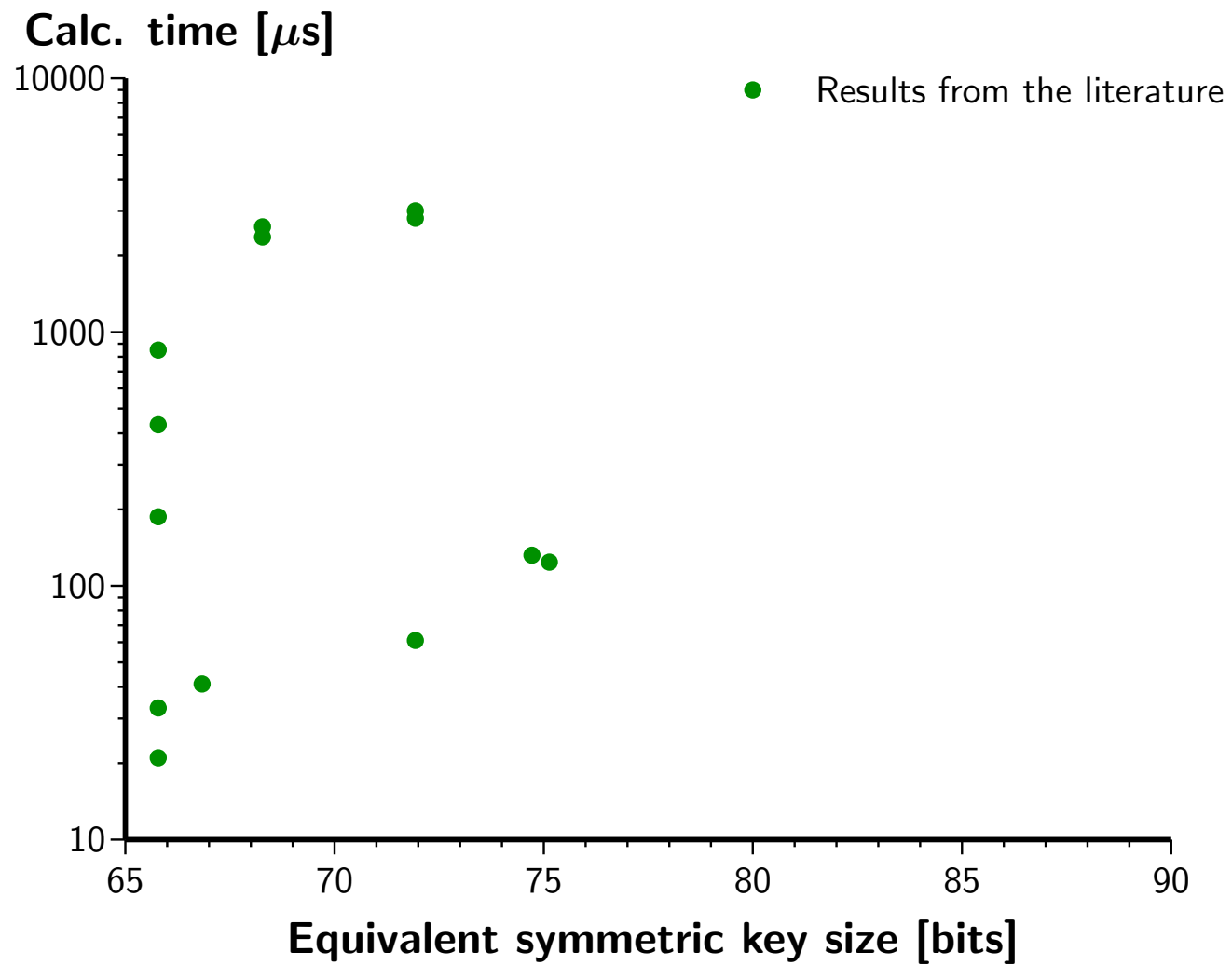
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- ▶ A first idea: an all-in-one unified operator:
 - shared resources
 - scalable architecture

The best area-time product of the literature...

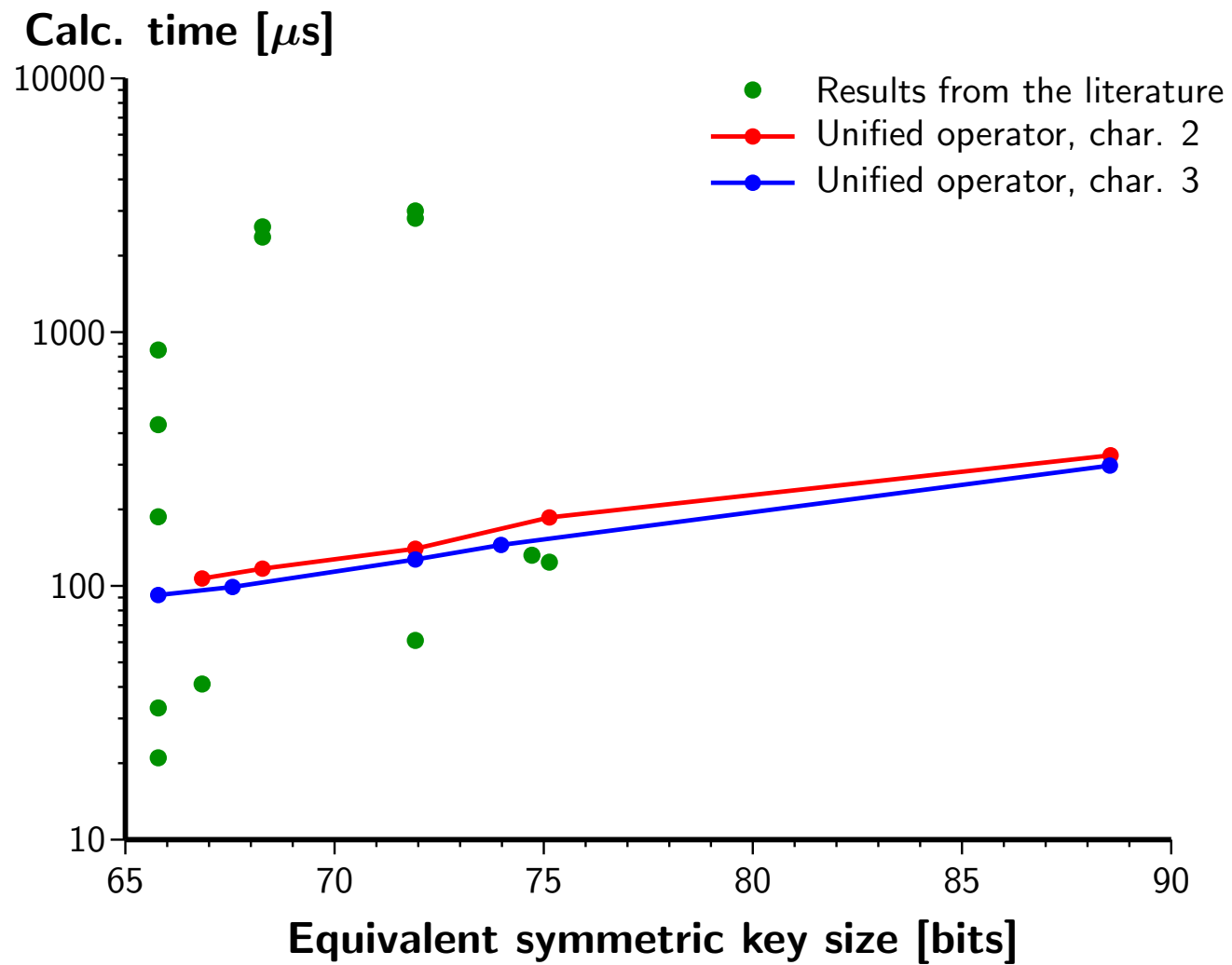


... But still quite slow



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(or not the fastest, at least!)



Motivations

- ▶ High speed is more important than low resources for some cryptographic applications
- ▶ Explore the other end of the area vs. time tradeoff:
 - faster but larger than the unified operator
 - what about the area-time product?

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- ▶ Explore the other end of the area vs. time tradeoff:
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 - what about the area-time product?
- ▶ Accelerate the computation by extracting as much parallelism as possible...
- ▶ ... Without dramatically increasing the resource requirements

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$$\hat{e}(P, Q) = \eta_T(P, Q)$$

- ▶ Computation of the η_T pairing
 - via Miller's algorithm: loop of $(m + 1)/2$ iterations
 - result only defined modulo N -th powers in $\mathbb{F}_{p^{km}}^\times$, with $N = \#E(\mathbb{F}_{p^m})$

Computation of the Tate pairing

- ▶ The Tate pairing over $E(\mathbb{F}_{p^m})$ is computed in two main steps

$$\hat{e}(P, Q) = \eta_T(P, Q)^M$$

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- ▶ Final exponentiation by $M = (p^{km} - 1)/N$

- required to obtain a unique value for each congruence class
- example in characteristic 3 ($k = 6$ and $N = 3^m + 1 \pm 3^{(m+1)/2}$):

$$M = \frac{3^{6m} - 1}{3^m + 1 \pm 3^{(m+1)/2}} = (3^{3m} - 1)(3^m + 1) \left(3^m + 1 \mp 3^{(m+1)/2}\right)$$

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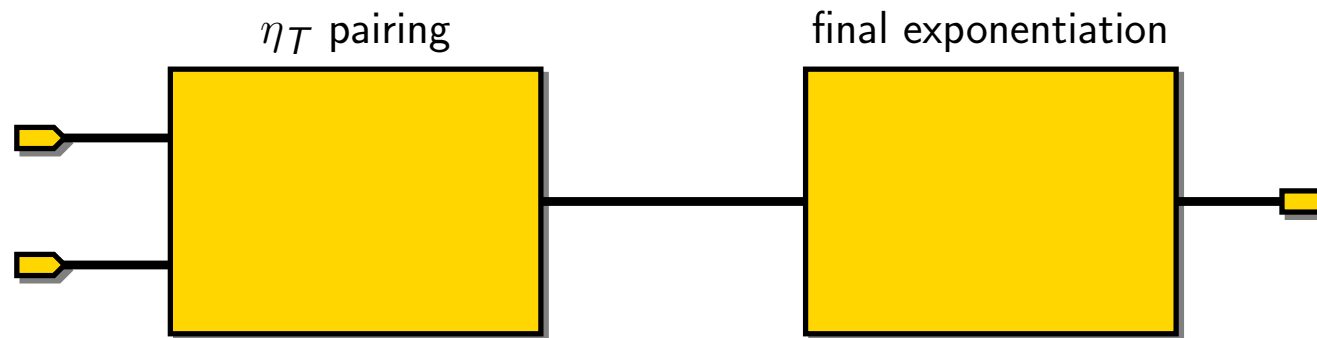
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- exploit the special form of the exponent: *ad-hoc* algorithm
- ▶ Two distinct computational requirements \Rightarrow use two distinct coprocessors

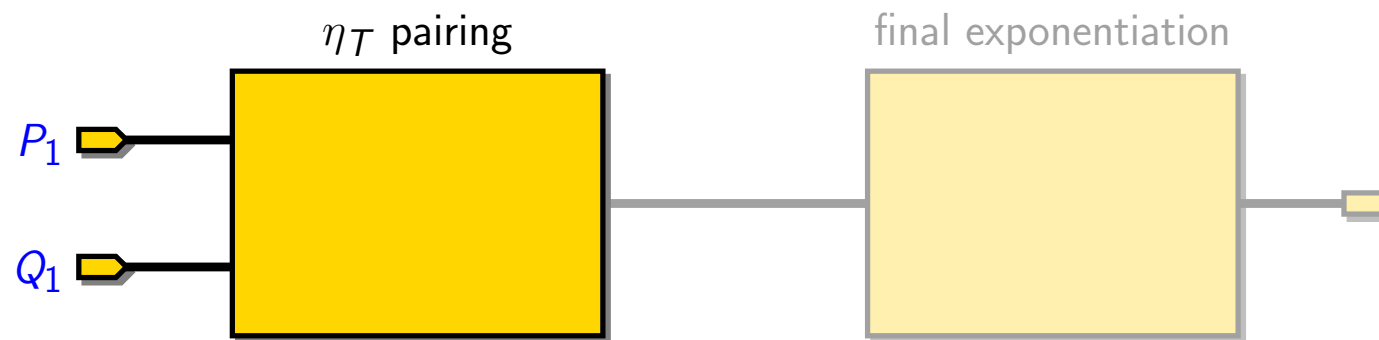
Two coprocessors for the Tate pairing

- ▶ The two operations are purely sequential



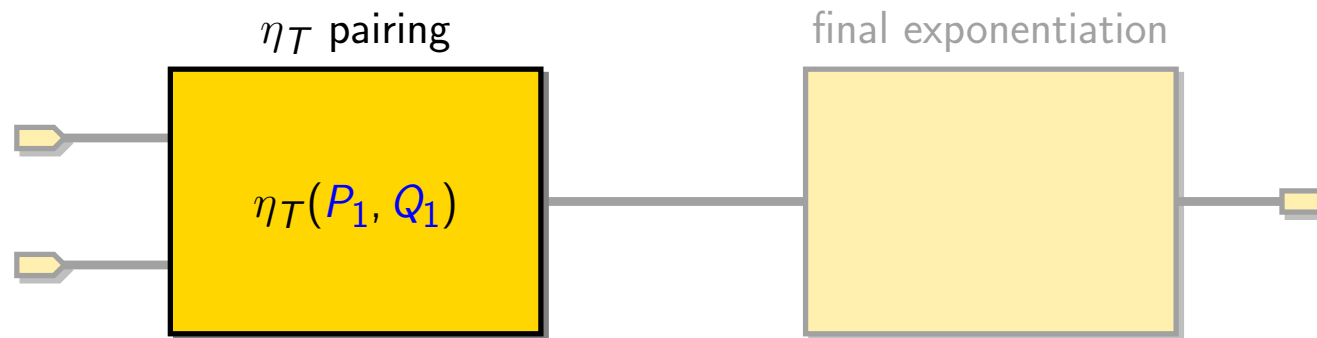
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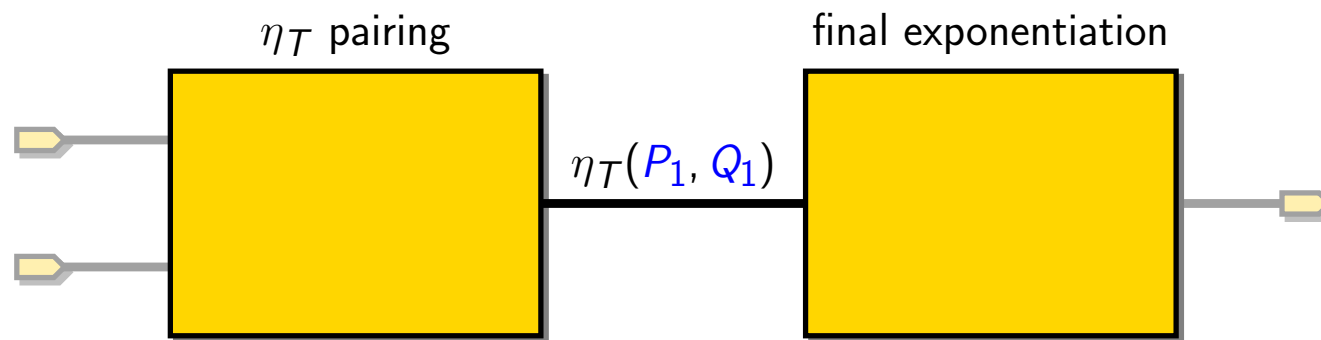
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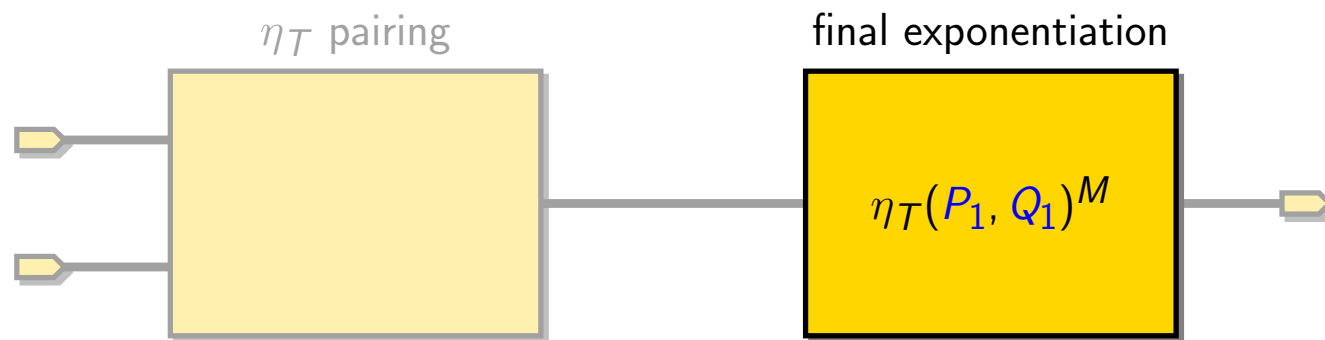
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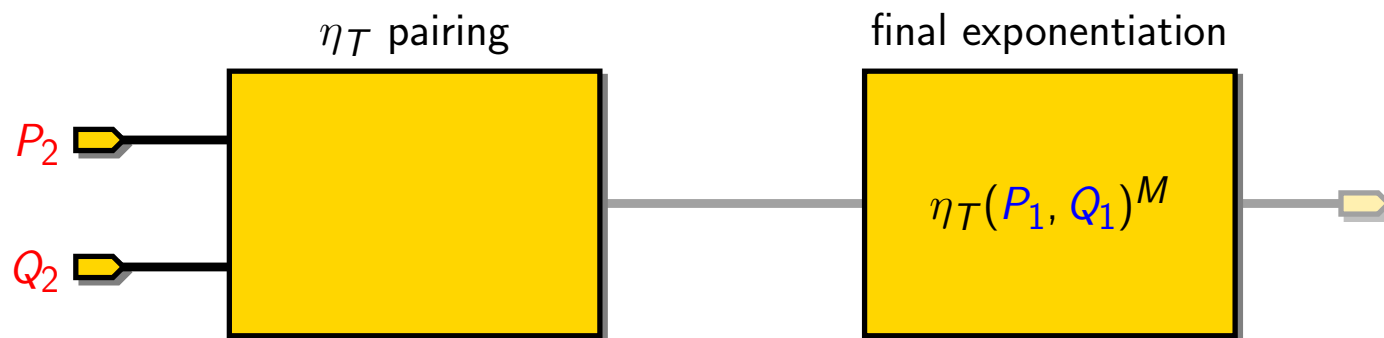
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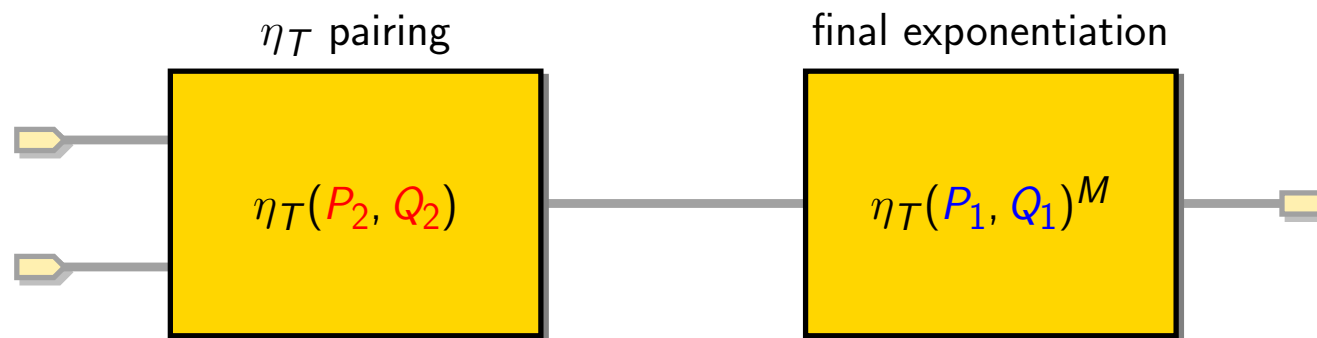
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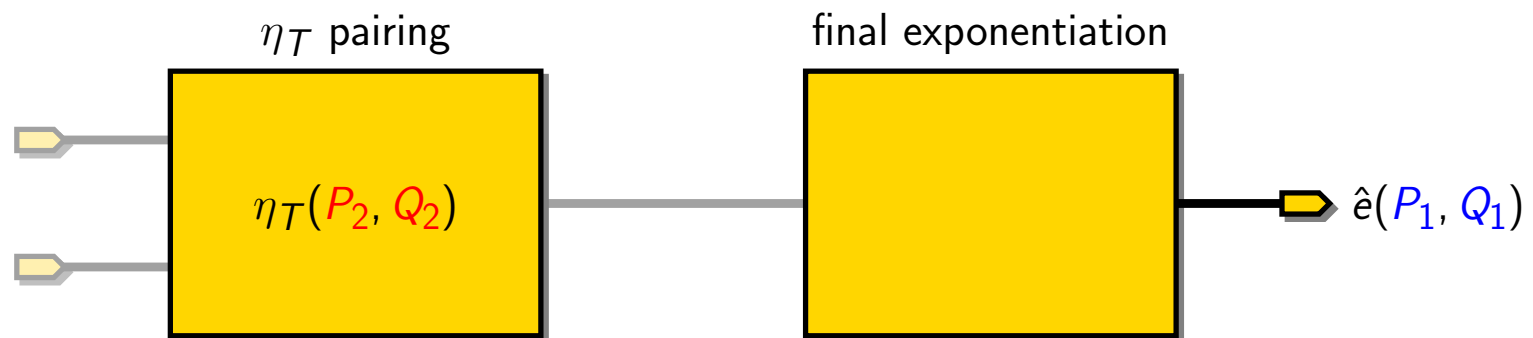
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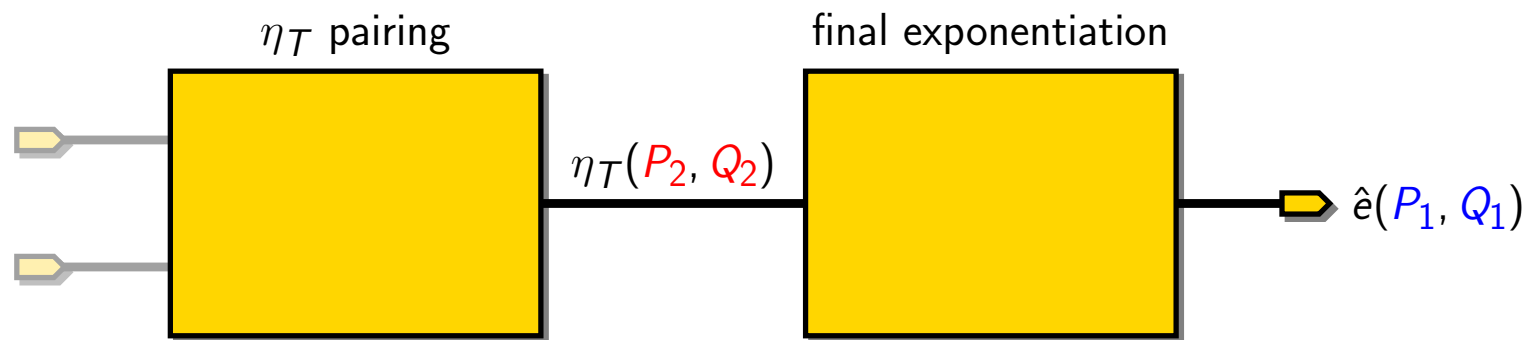
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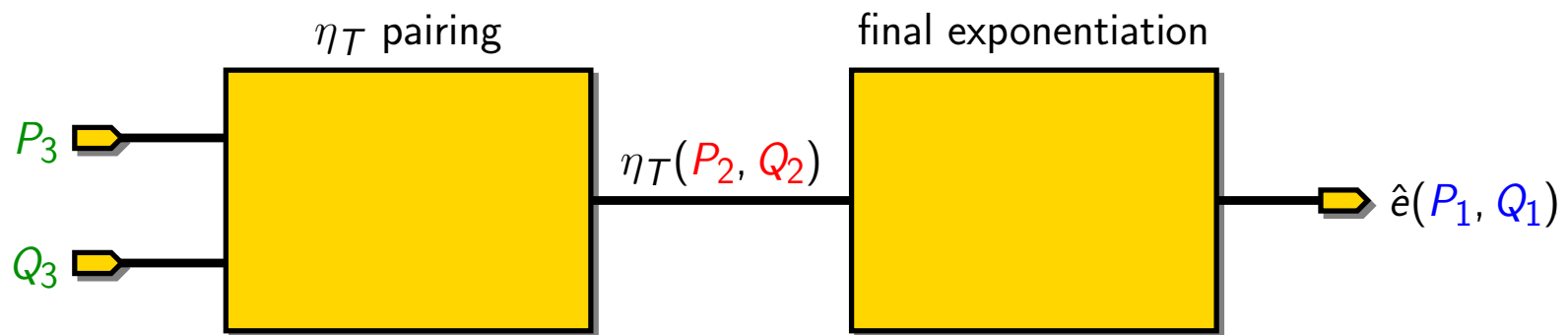
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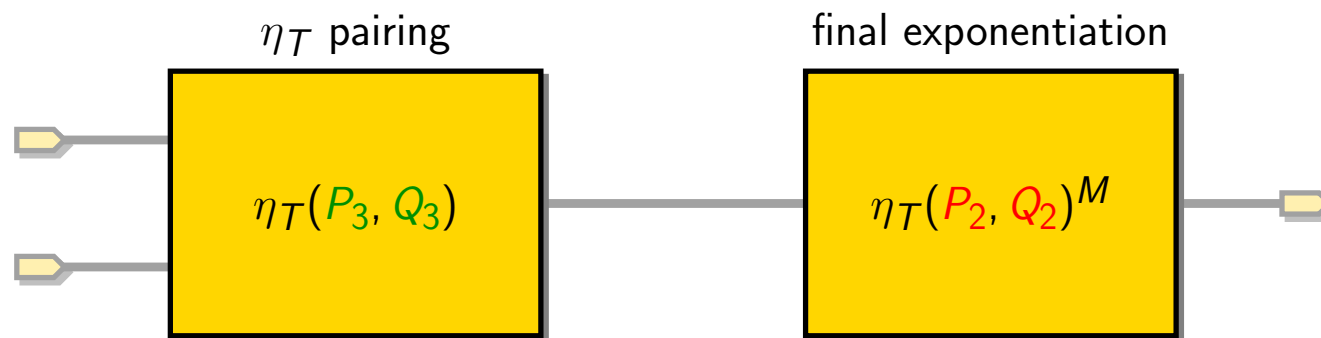
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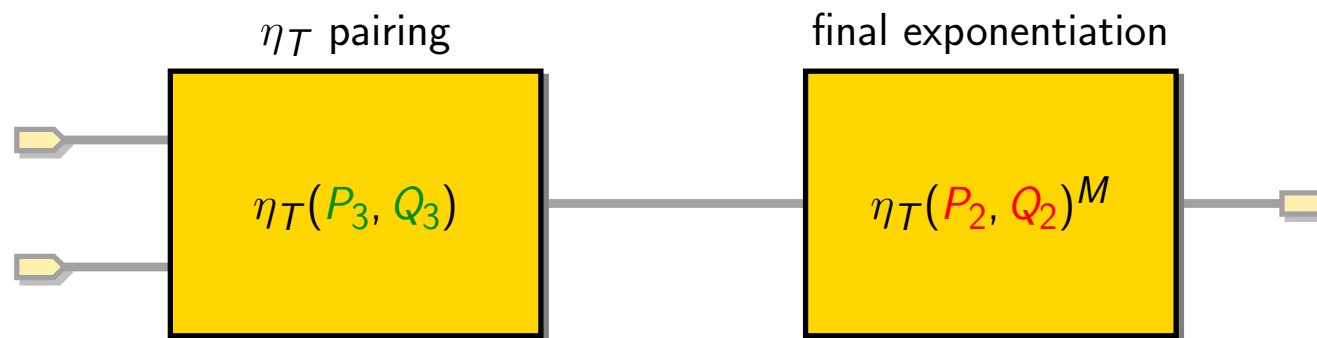
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 - higher throughput

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- ▶ Only one active coprocessor at every moment
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- ▶ Balance the computation time between the two coprocessors

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η_T pairing algorithm

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$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

$$x_Q \leftarrow x_P^9 \pm 1 ; y_Q \leftarrow -y_P^9$$

$$t \leftarrow x_P + x_Q ; u \leftarrow y_P y_Q$$

$$S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$$

$$R \leftarrow R \cdot S$$

$$R \leftarrow R^3$$

end for

► Four tasks per iteration:

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► Four tasks per iteration:

- ① update the coordinates
- ② compute the line equation
- ③ accumulate the new factor
- ④ cube the partial product

η_T pairing algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

① $x_Q \leftarrow x_Q^9 \pm 1 ; y_Q \leftarrow -y_Q^9$

4 Frobenius, 2 + (\mathbb{F}_{3^m})

② $t \leftarrow x_P + x_Q ; u \leftarrow y_P y_Q$
 $S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

④ $R \leftarrow R^3$

end for

► Four tasks per iteration:

- ① update the coordinates
- ② compute the line equation
- ③ accumulate the new factor
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 $S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$ $2 \times, 1 +$ (\mathbb{F}_{3^m})

③ $R \leftarrow R \cdot S$ $1 \times$ $(\mathbb{F}_{3^{6m}})$

④ $R \leftarrow R^3$ 1 Frobenius $(\mathbb{F}_{3^{6m}})$

end for

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 $S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$ $2 \times, 1 + (\mathbb{F}_{3^m})$

③ $R \leftarrow R \cdot S$ $1 \times (\mathbb{F}_{3^{6m}})$

④ $R \leftarrow R^3$ 6 Frobenius, $6 + (\mathbb{F}_{3^m})$

end for

► Four tasks per iteration:

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③ $R \leftarrow R \cdot S$ $12 \times, 59 + (\mathbb{F}_{3^m})$

④ $R \leftarrow R^3$ 6 Frobenius, 6 + (\mathbb{F}_{3^m})

end for

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 $S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$ $2 \times, 1 + (\mathbb{F}_{3^m})$

③ $R \leftarrow R \cdot S$ $15 \times, 29 + (\mathbb{F}_{3^m})$

④ $R \leftarrow R^3$ 6 Frobenius, 6 + (\mathbb{F}_{3^m})

end for

► Four tasks per iteration:

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 $S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$ 2 \times , 1 + (\mathbb{F}_{3^m})

③ $R \leftarrow R \cdot S$ 15 \times , 29 + (\mathbb{F}_{3^m})

④ $R \leftarrow R^3$ 6 Frobenius, 6 + (\mathbb{F}_{3^m})

end for

► Four tasks per iteration:

- ① update the coordinates
- ② compute the line equation
- ③ accumulate the new factor
- ④ cube the partial product

► Total cost: 17 \times , 10 Frobenius and 38 + over \mathbb{F}_{3^m}

Accelerating the η_T pairing

- ▶ Total cost: $17 \times$, 10 Frobenius and $38 +$ over \mathbb{F}_{3^m} per iteration

Accelerating the η_T pairing

- ▶ Total cost: $17 \times$, 10 Frobenius and $38 +$ over \mathbb{F}_{3^m} per iteration
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 - critical operation: \times

Accelerating the η_T pairing

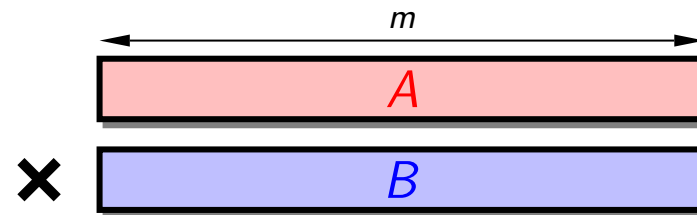
- ▶ Total cost: $17 \times$, 10 Frobenius and $38 +$ over \mathbb{F}_{3^m} per iteration
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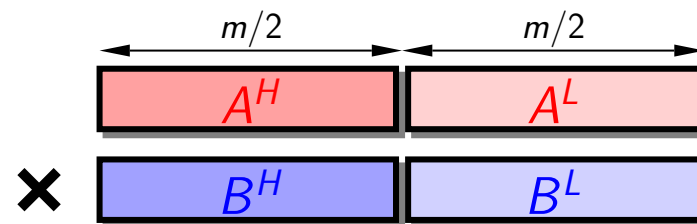
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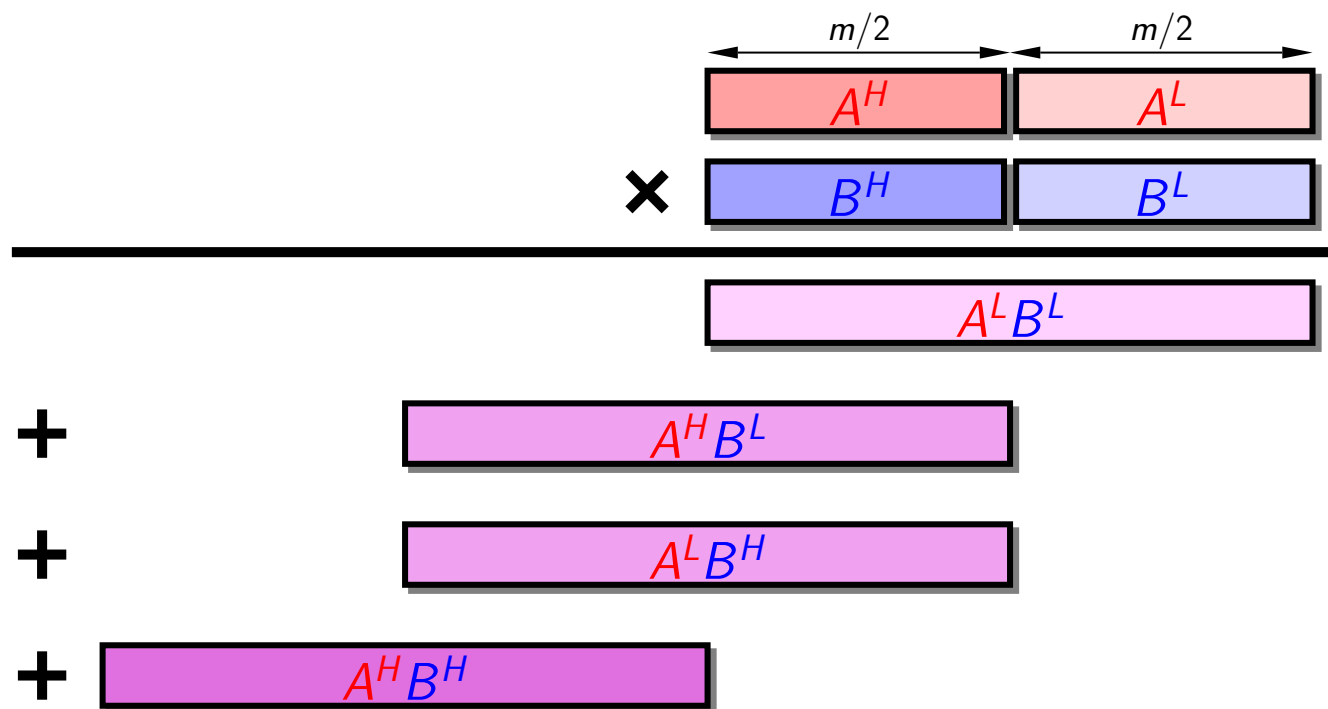
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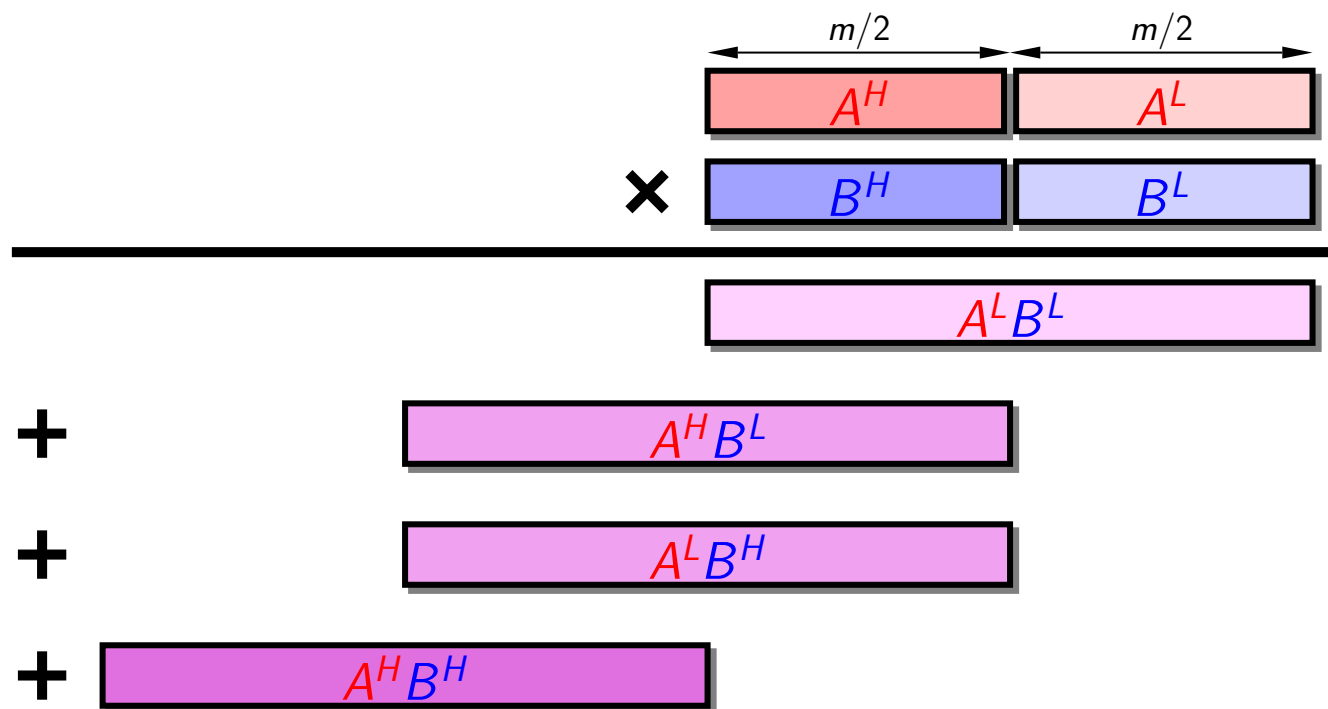
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Accelerating the η_T pairing

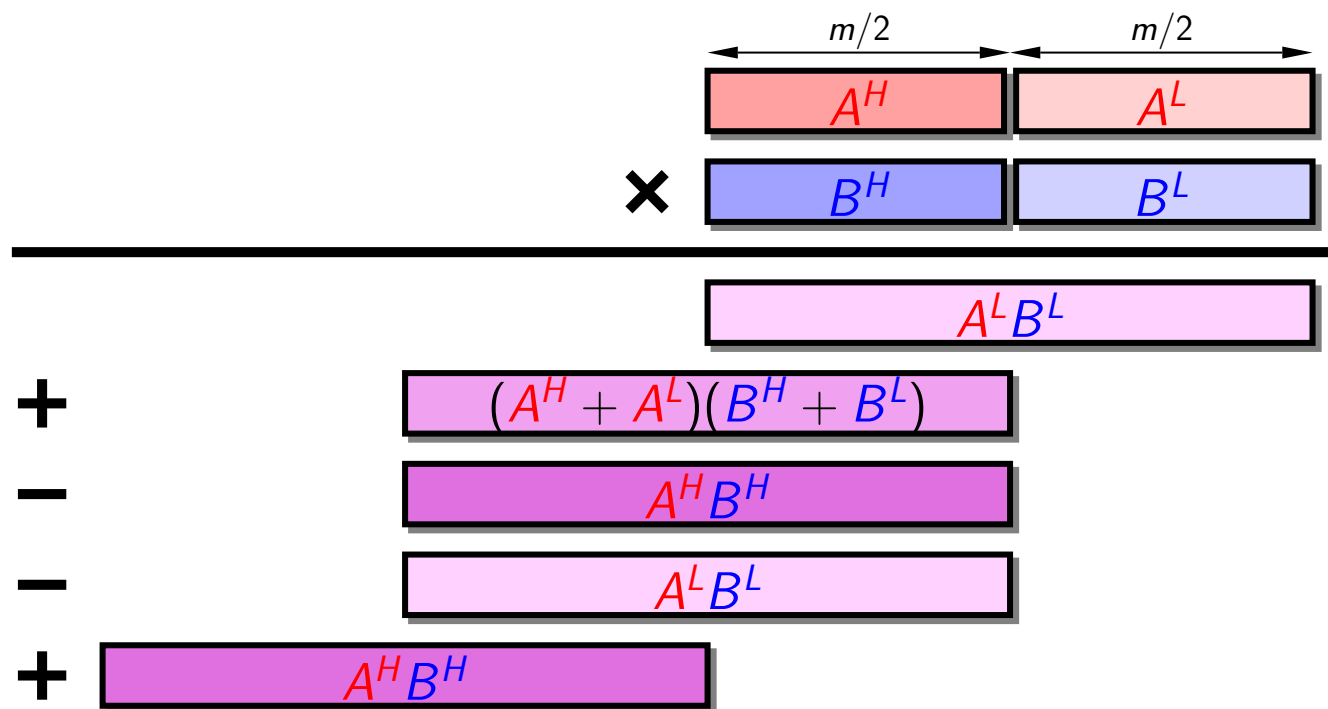
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$$A^H B^L + A^L B^H = (A^H + A^L)(B^H + B^L) - A^H B^H - A^L B^L$$

Accelerating the η_T pairing

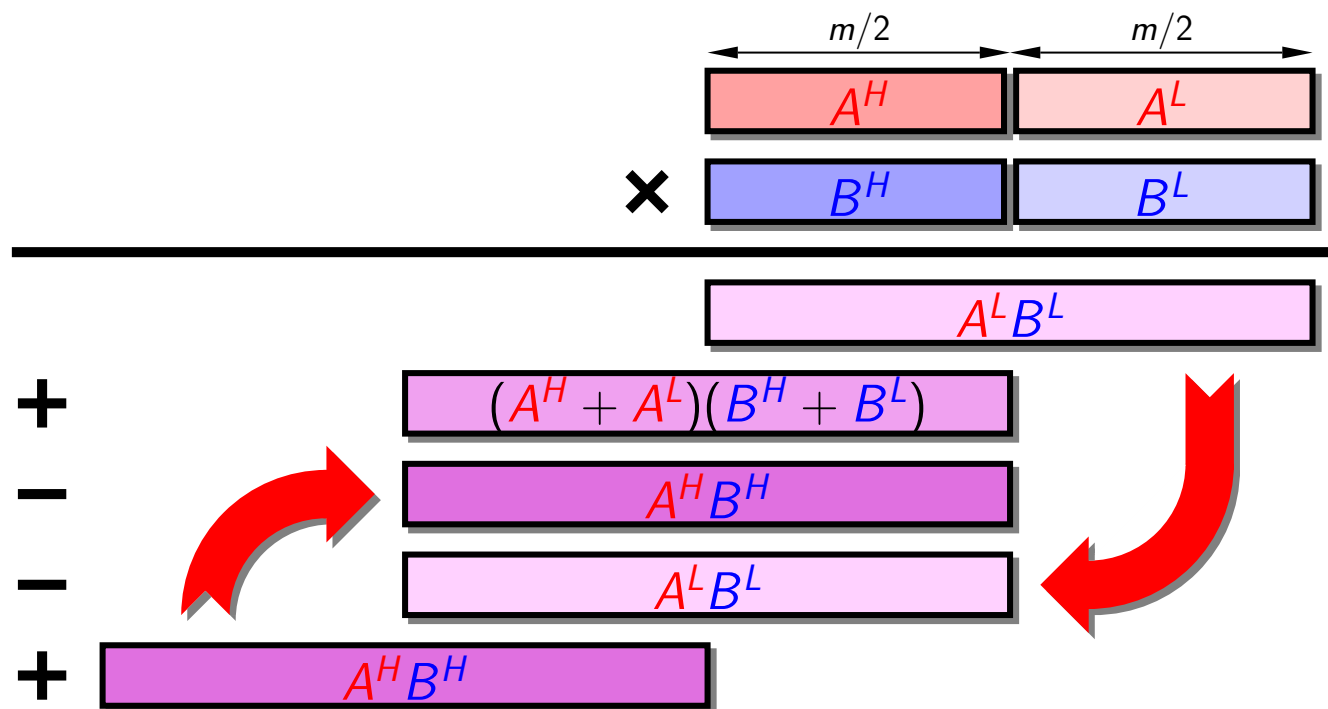
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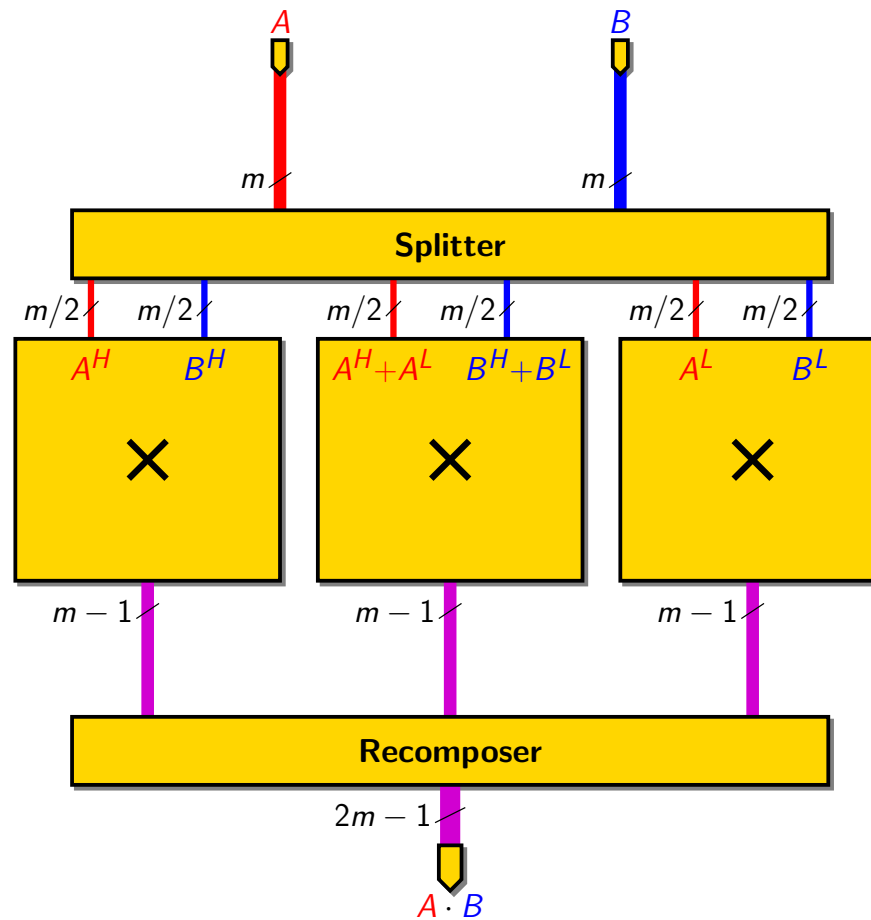
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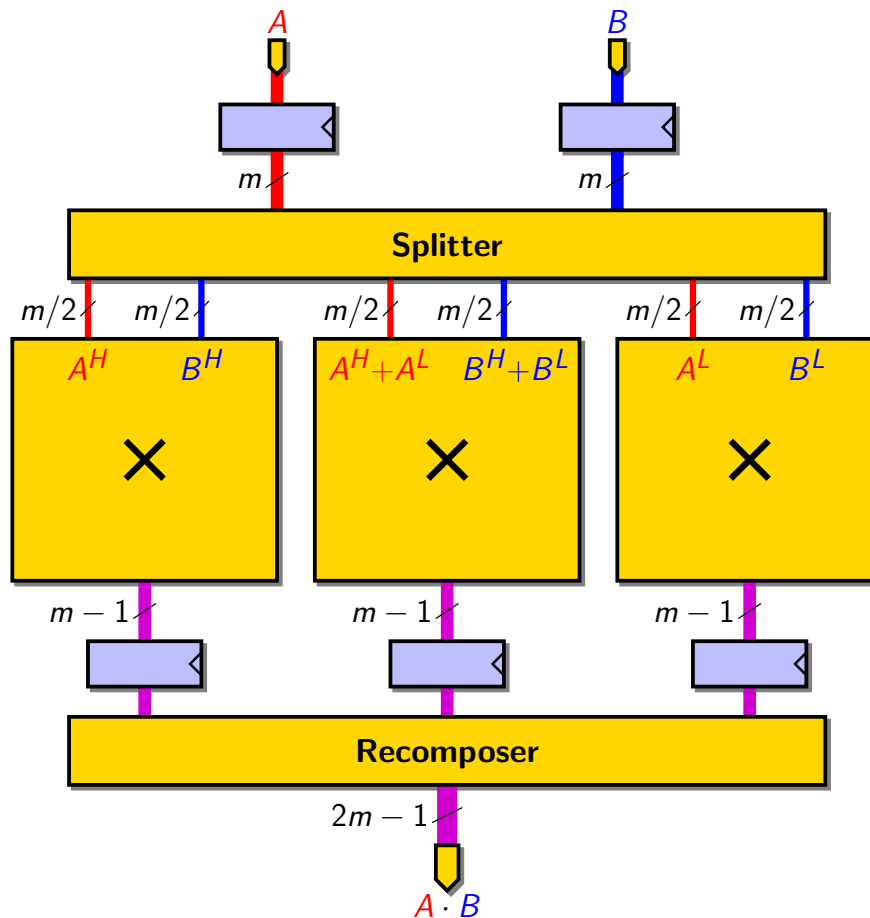
$$A^H B^L + A^L B^H = (A^H + A^L)(B^H + B^L) - A^H B^H - A^L B^L$$

A parallel Karatsuba-Ofman multiplier



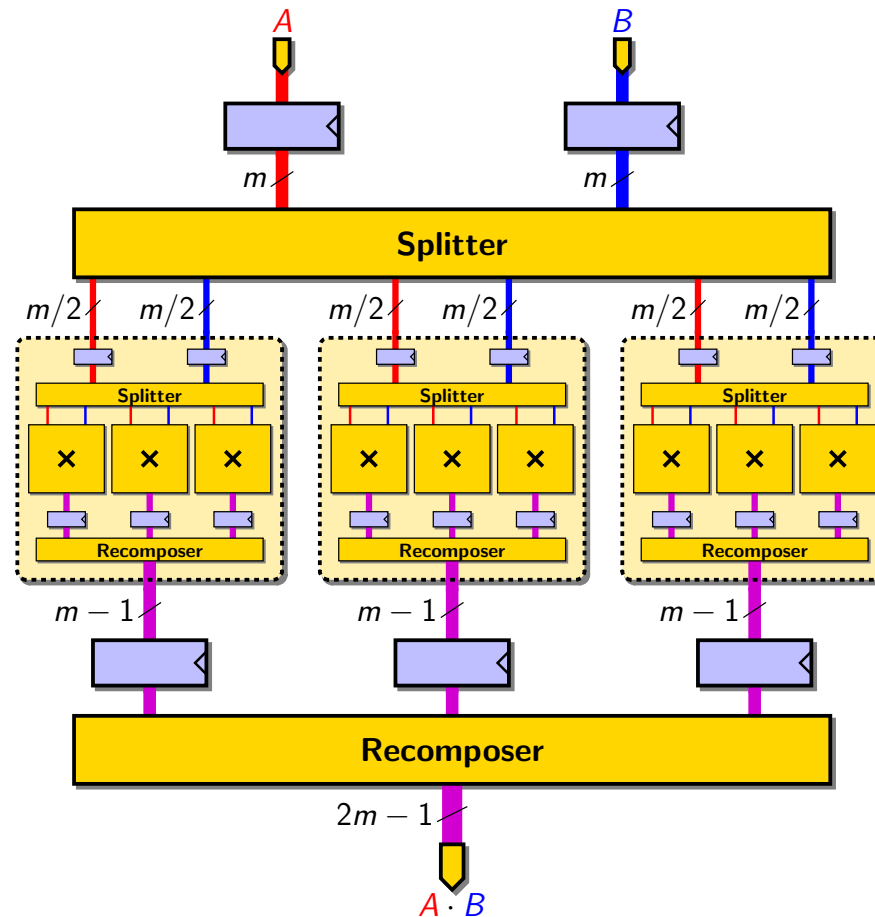
- **fully parallel:** all sub-products are computed in parallel

A parallel Karatsuba-Ofman multiplier



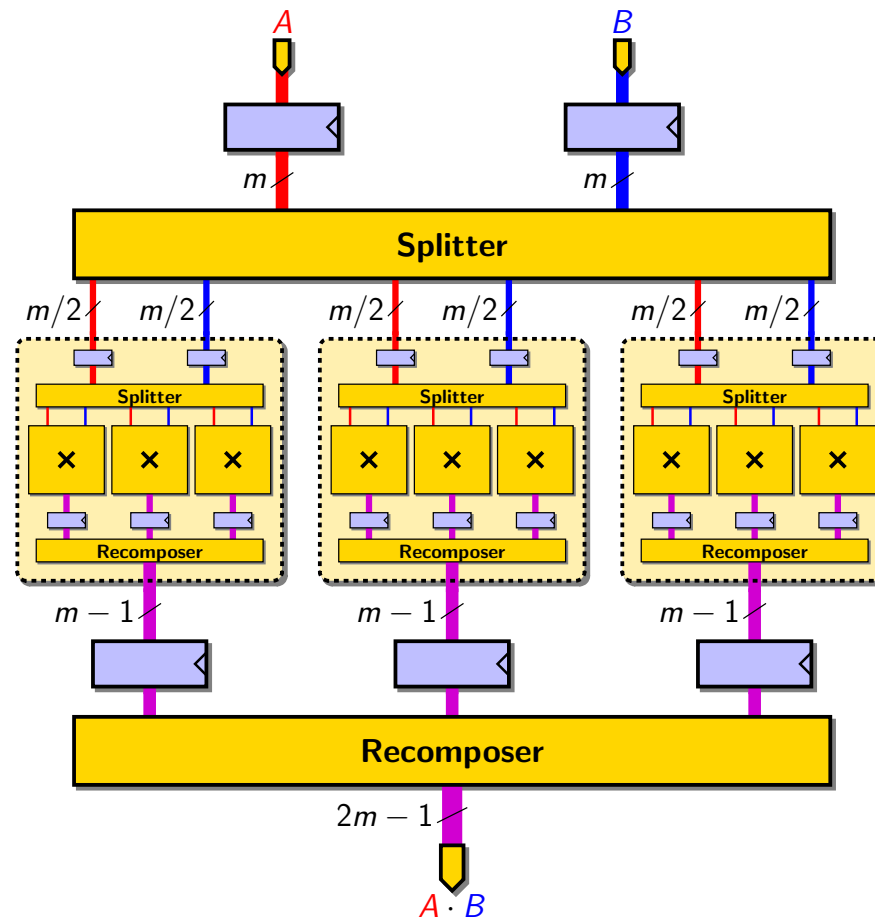
- fully parallel: all sub-products are computed in parallel
- pipelined architecture: higher clock frequency, one product per cycle

A parallel Karatsuba-Ofman multiplier



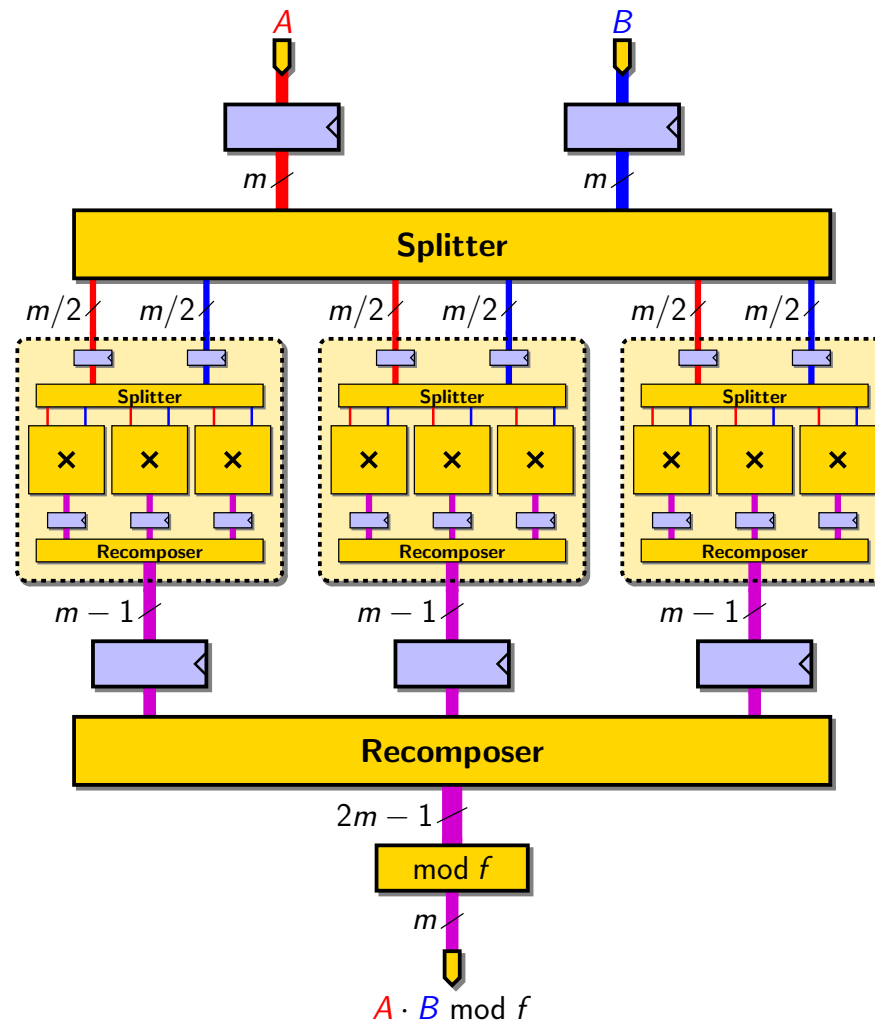
- fully parallel: all sub-products are computed in parallel
- pipelined architecture: higher clock frequency, one product per cycle
- sub-products recursively implemented as Karatsuba-Ofman multipliers

A parallel Karatsuba-Ofman multiplier



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- support for other variants: odd-even split, 3-way split, ...

A parallel Karatsuba-Ofman multiplier



- fully parallel: all sub-products are computed in parallel
- pipelined architecture: higher clock frequency, one product per cycle
- sub-products recursively implemented as Karatsuba-Ofman multipliers
- support for other variants: odd-even split, 3-way split, ...
- final reduction modulo the irreducible polynomial f

Accelerating the η_T pairing

- ▶ Total cost: $17 \times$, 10 Frobenius and $38 +$ over \mathbb{F}_{3^m} per iteration
- ▶ η_T coprocessor based on a single large multiplier:
 - parallel Karatsuba-Ofman architecture
 - 7-stage pipeline
 - one product per cycle

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- ▶ η_T coprocessor based on a single large multiplier:
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 - 7-stage pipeline
 - one product per cycle
- ▶ Challenge: keep the multiplier busy at all times
- ▶ Careful scheduling to avoid pipeline bubbles (idle cycles):
 - ensure that multiplication operands are always available
 - avoid memory congestion issues

Data dependencies

$$\textcircled{1} \quad x_Q \leftarrow x_Q^g \pm 1 ; y_Q \leftarrow -y_Q^g$$

$$\textcircled{2} \quad \begin{aligned} t &\leftarrow x_P + x_Q ; u \leftarrow y_P y_Q \\ S &\leftarrow -t^2 + u\sigma - t\rho - \rho^2 \end{aligned}$$

$$\textcircled{3} \quad R \leftarrow R \cdot S$$

$$\textcircled{4} \quad R \leftarrow R^3$$

Data dependencies

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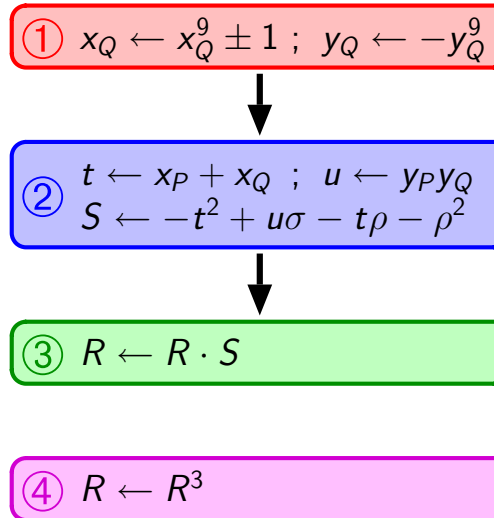


$$\textcircled{2} \quad \begin{aligned} t &\leftarrow x_P + x_Q ; u \leftarrow y_P y_Q \\ S &\leftarrow -t^2 + u\sigma - t\rho - \rho^2 \end{aligned}$$

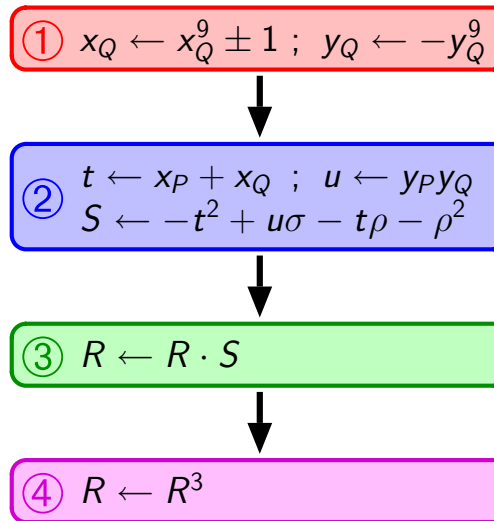
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Data dependencies

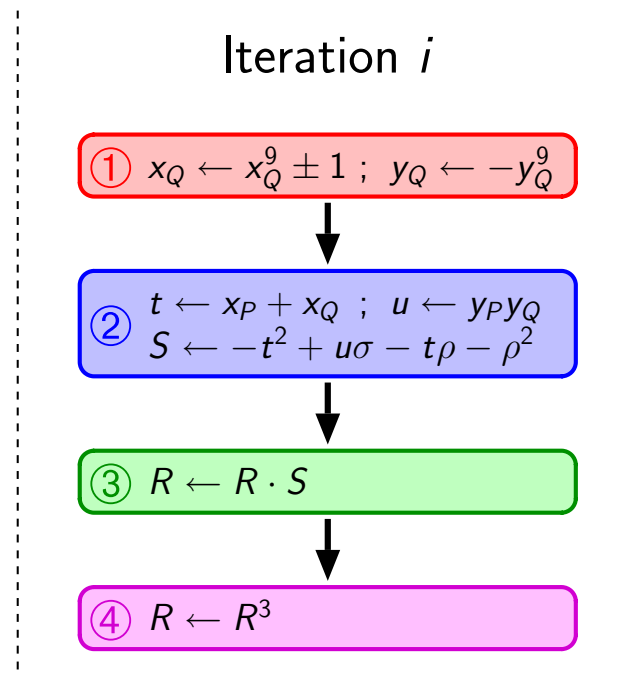


Data dependencies



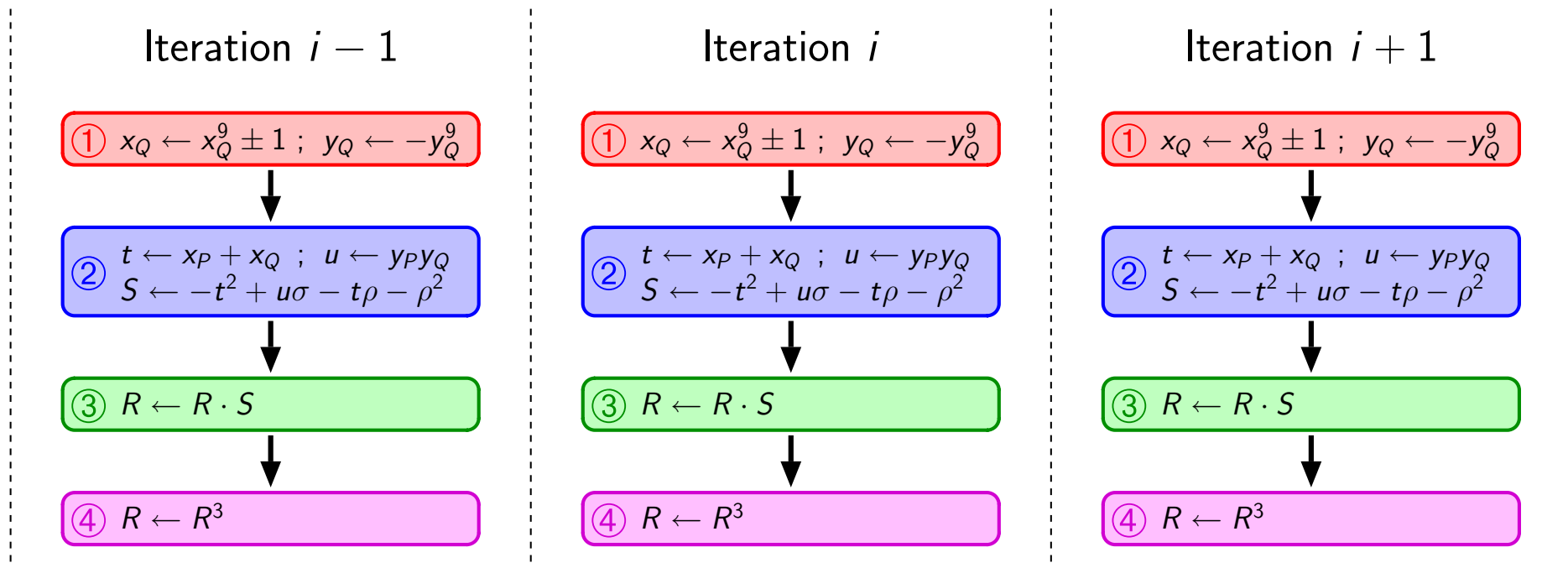
- Sequential dependencies between the tasks

Data dependencies



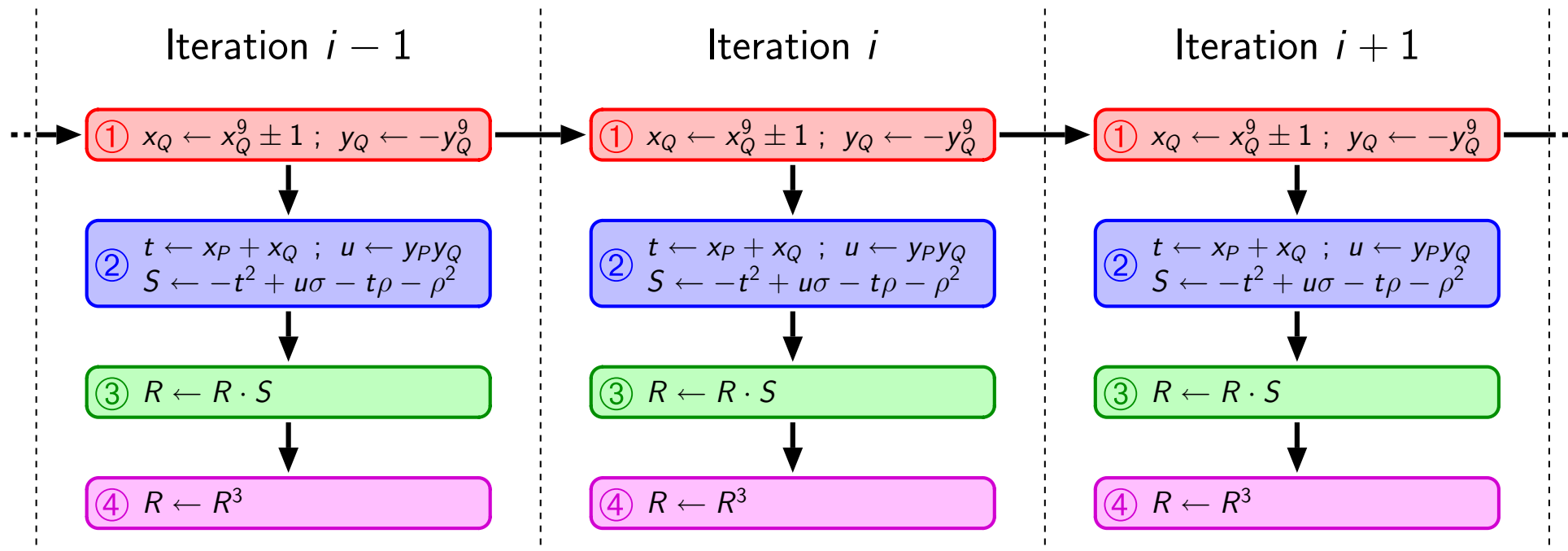
- ▶ Sequential dependencies between the tasks in each iteration

Data dependencies



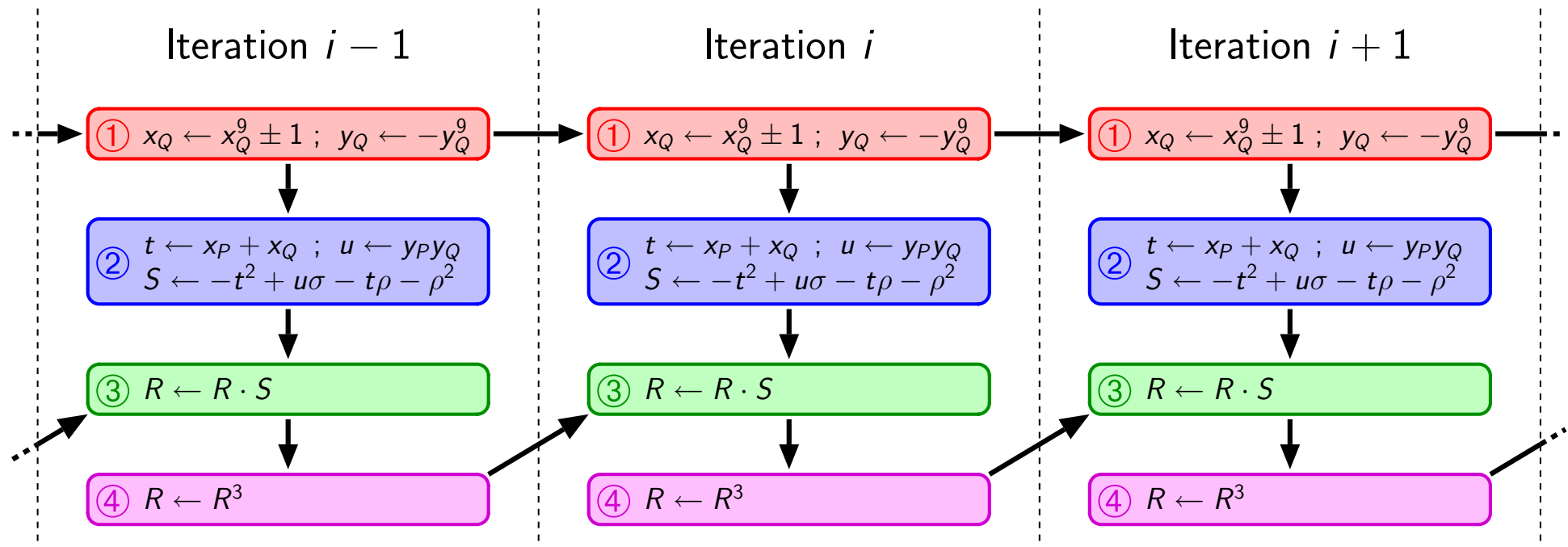
- Sequential dependencies between the tasks in each iteration

Data dependencies



- ▶ Sequential dependencies between the tasks in each iteration
- ▶ Dependencies between consecutive iterations

Data dependencies



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- ▶ Dependencies between consecutive iterations

Task scheduling



Task scheduling

$$\textcircled{1} \quad x_Q \leftarrow x_Q^9 \pm 1 ; y_Q \leftarrow -y_Q^9$$

Task scheduling

① 4 Frobenius, 2 +

Task scheduling



Task scheduling

①

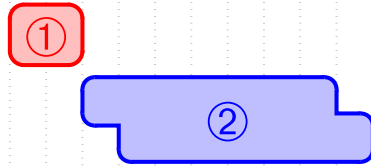
$$\begin{aligned} \textcircled{2} \quad & t \leftarrow x_P + x_Q ; u \leftarrow y_P y_Q \\ & S \leftarrow -t^2 + u\sigma - t\rho - \rho^2 \end{aligned}$$

Task scheduling

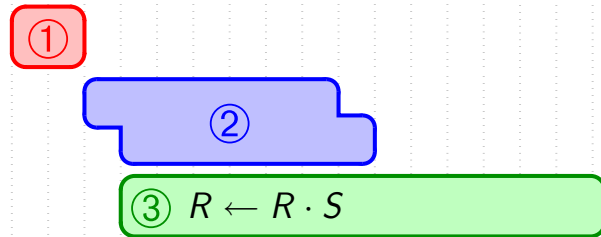
①

② $2 \times, 1 +$

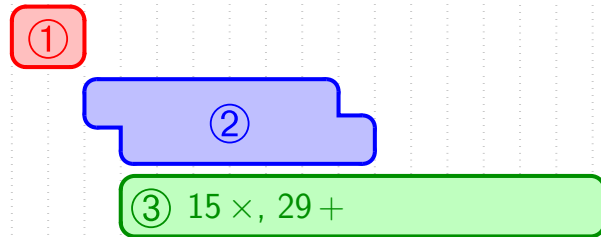
Task scheduling



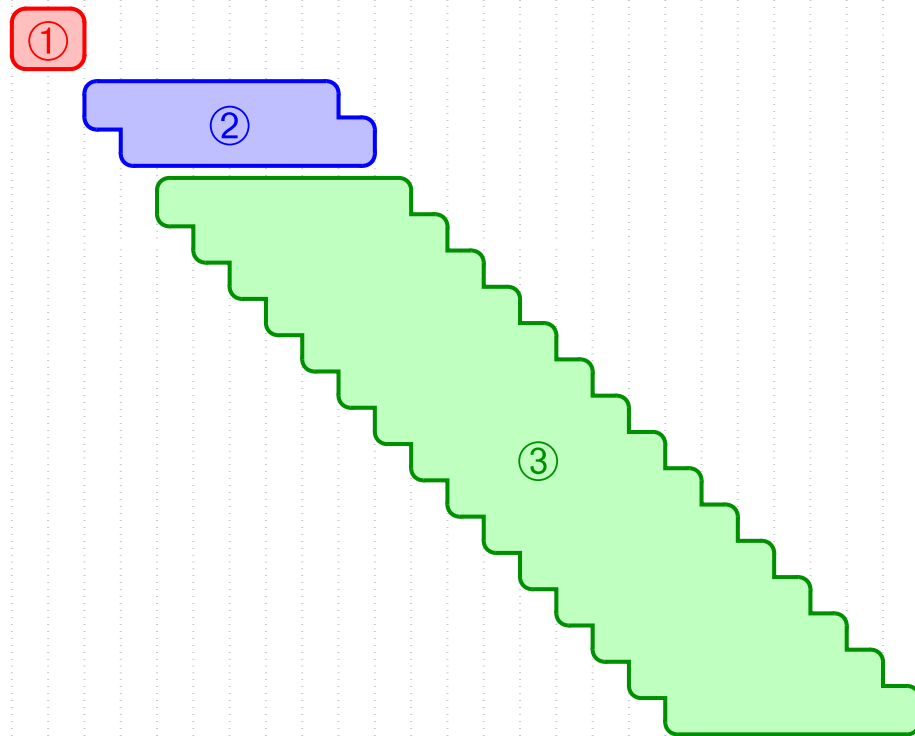
Task scheduling



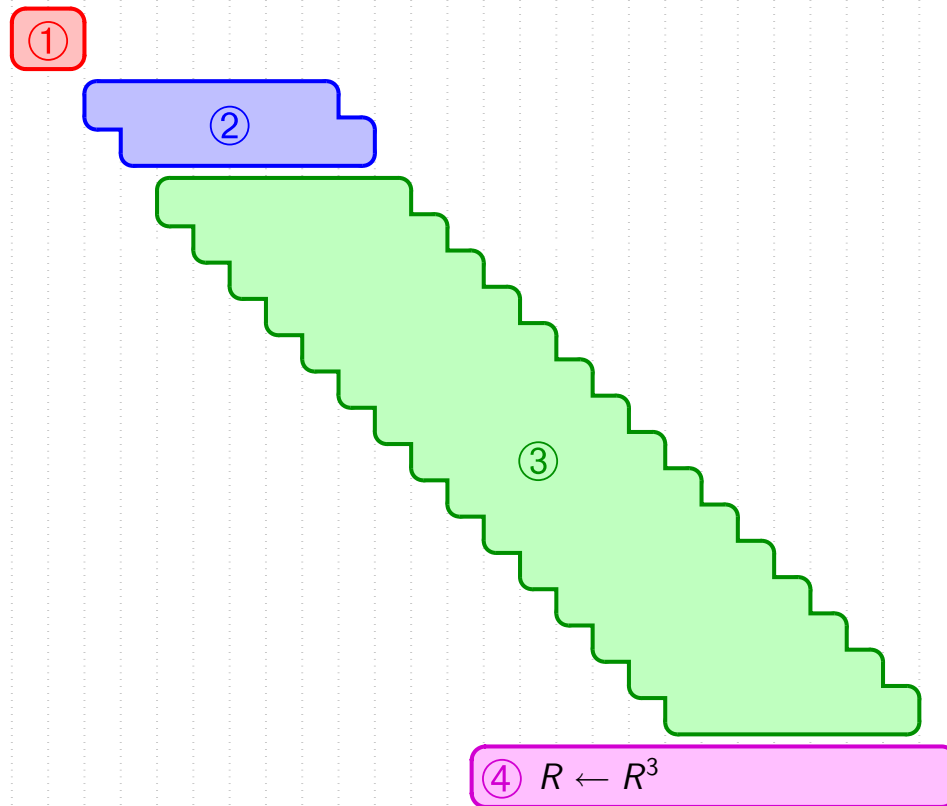
Task scheduling



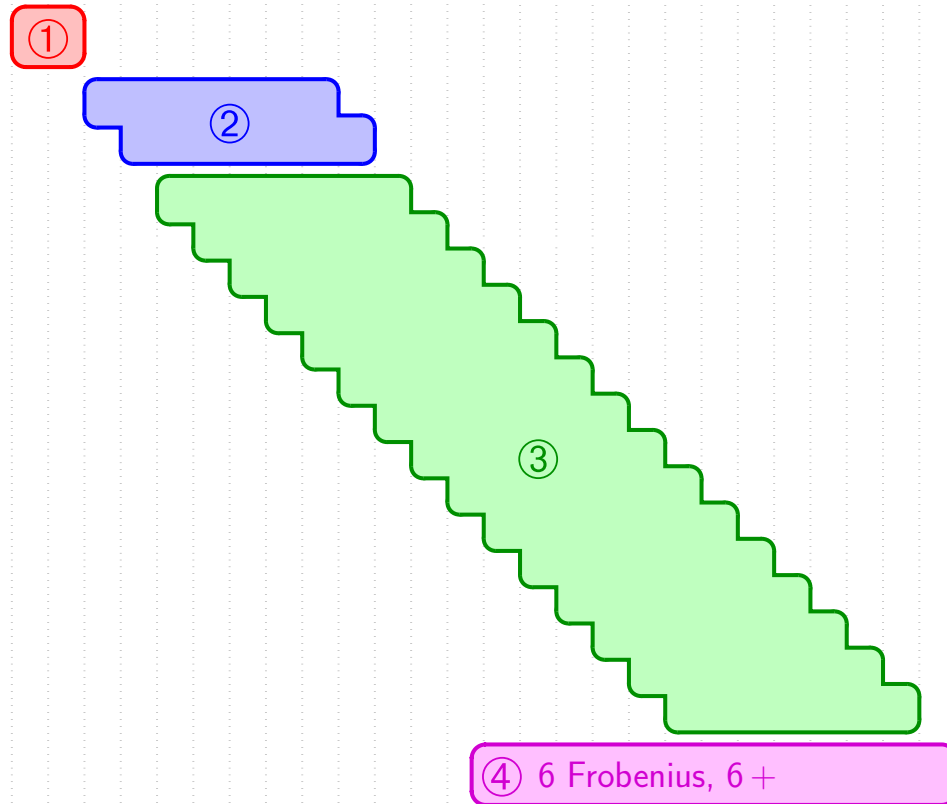
Task scheduling



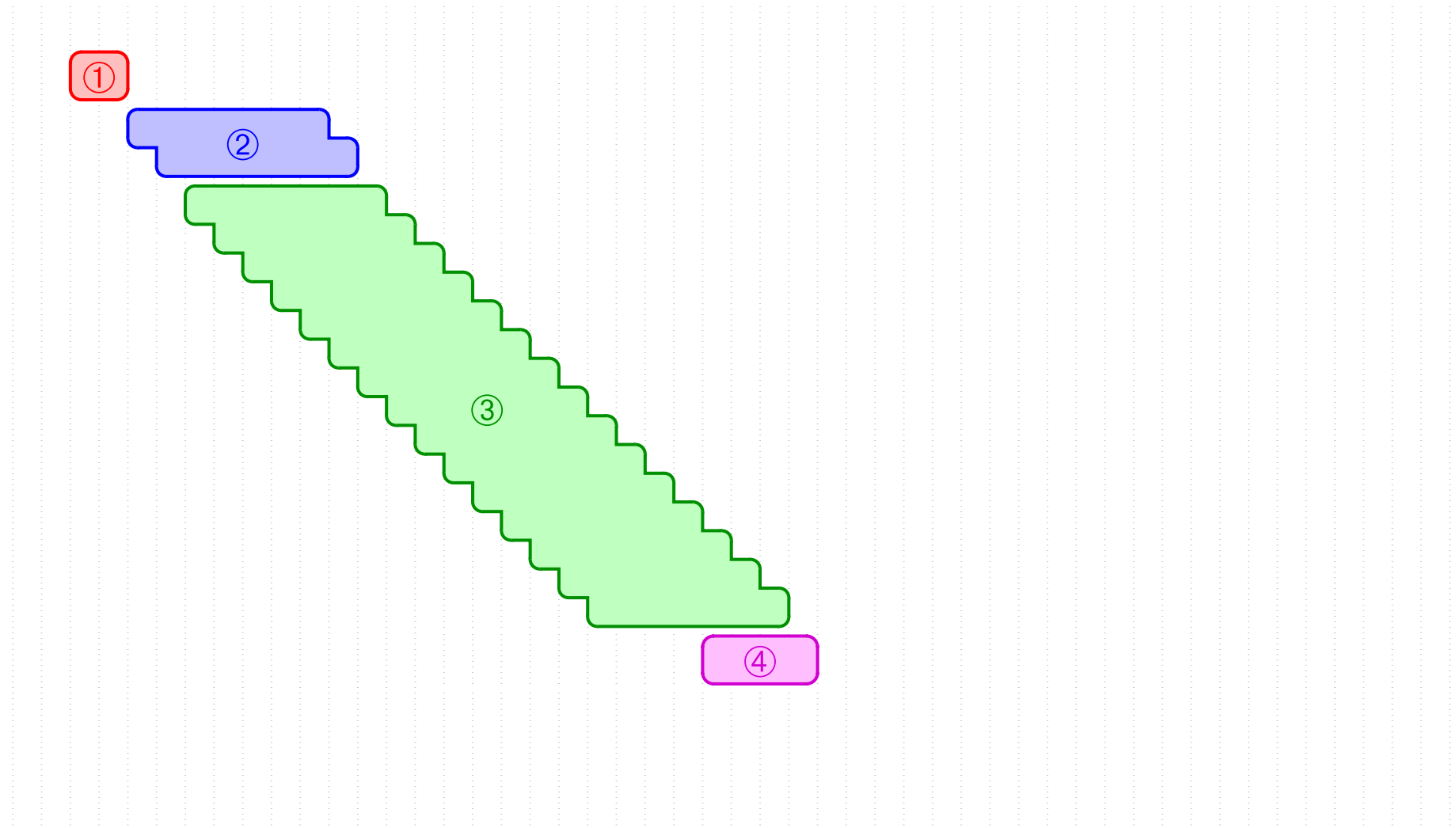
Task scheduling



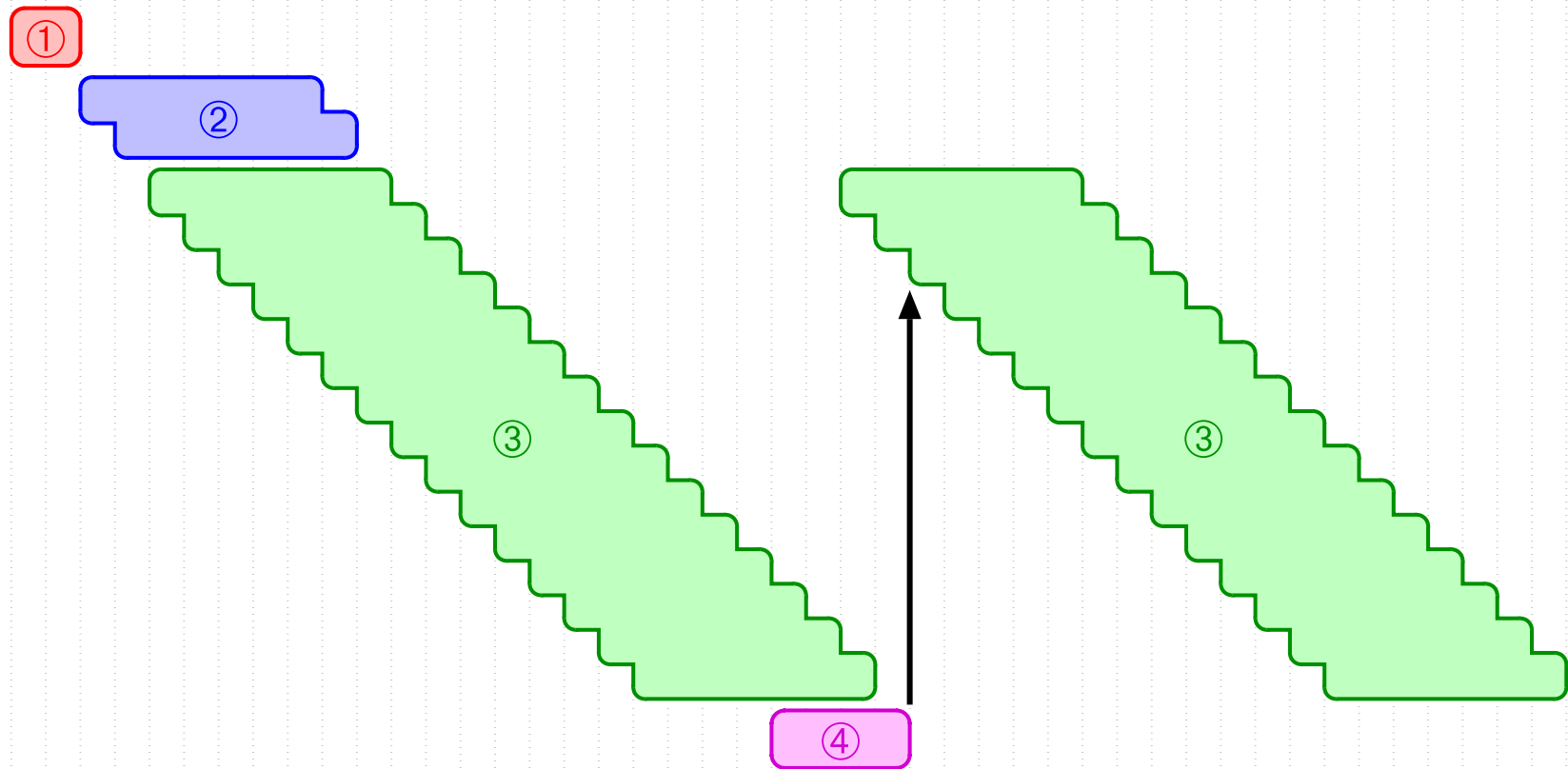
Task scheduling



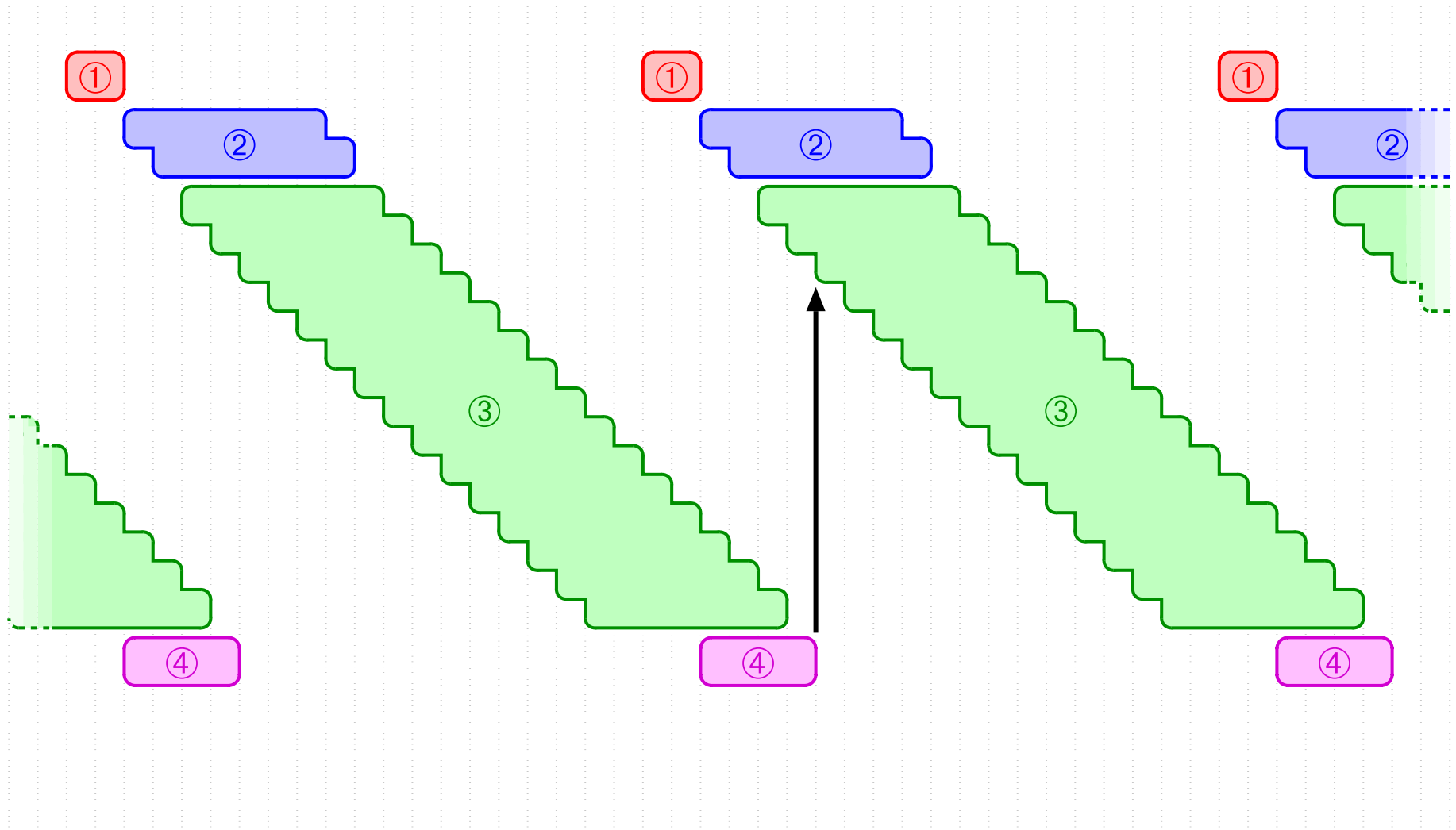
Task scheduling



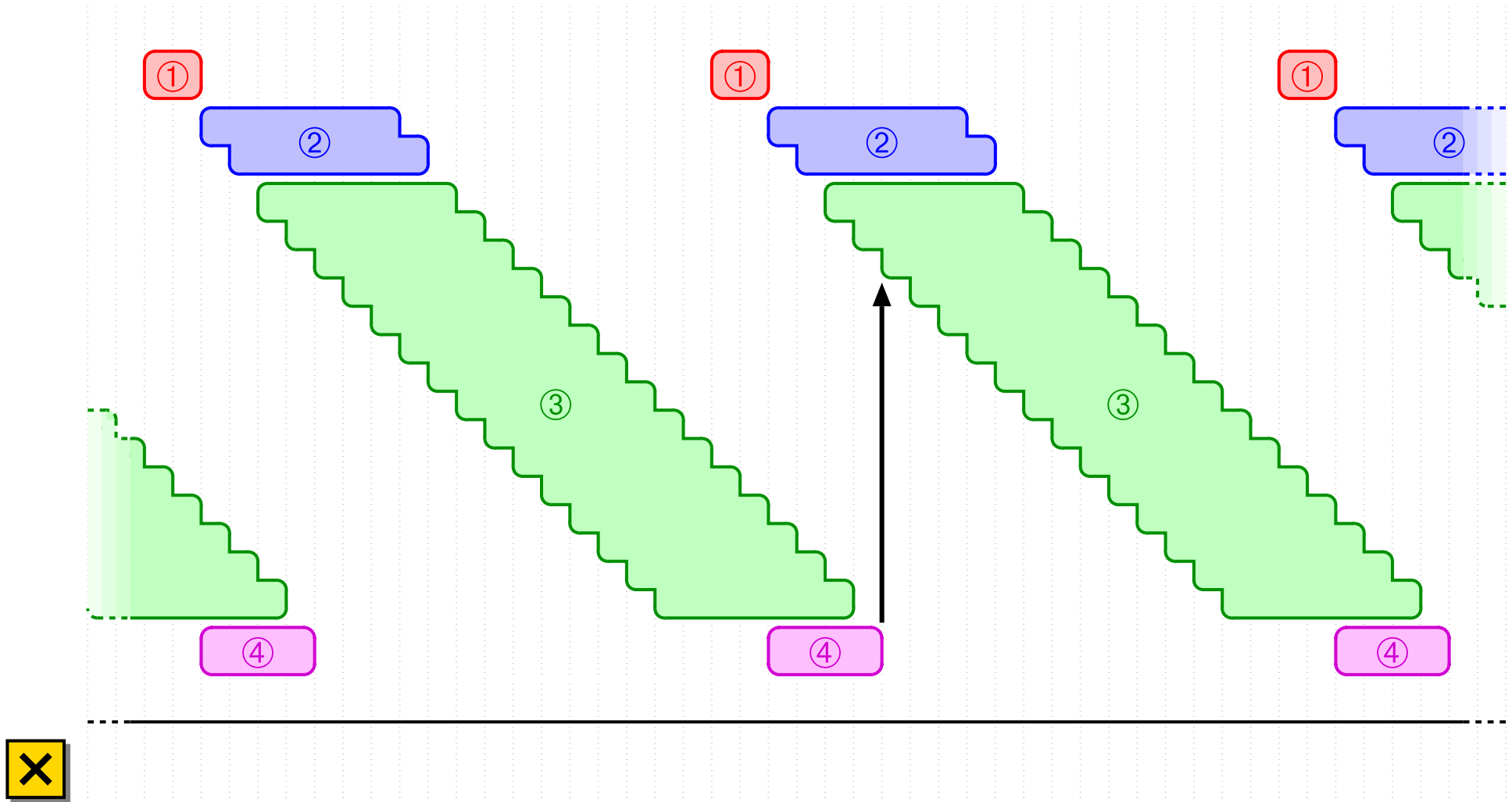
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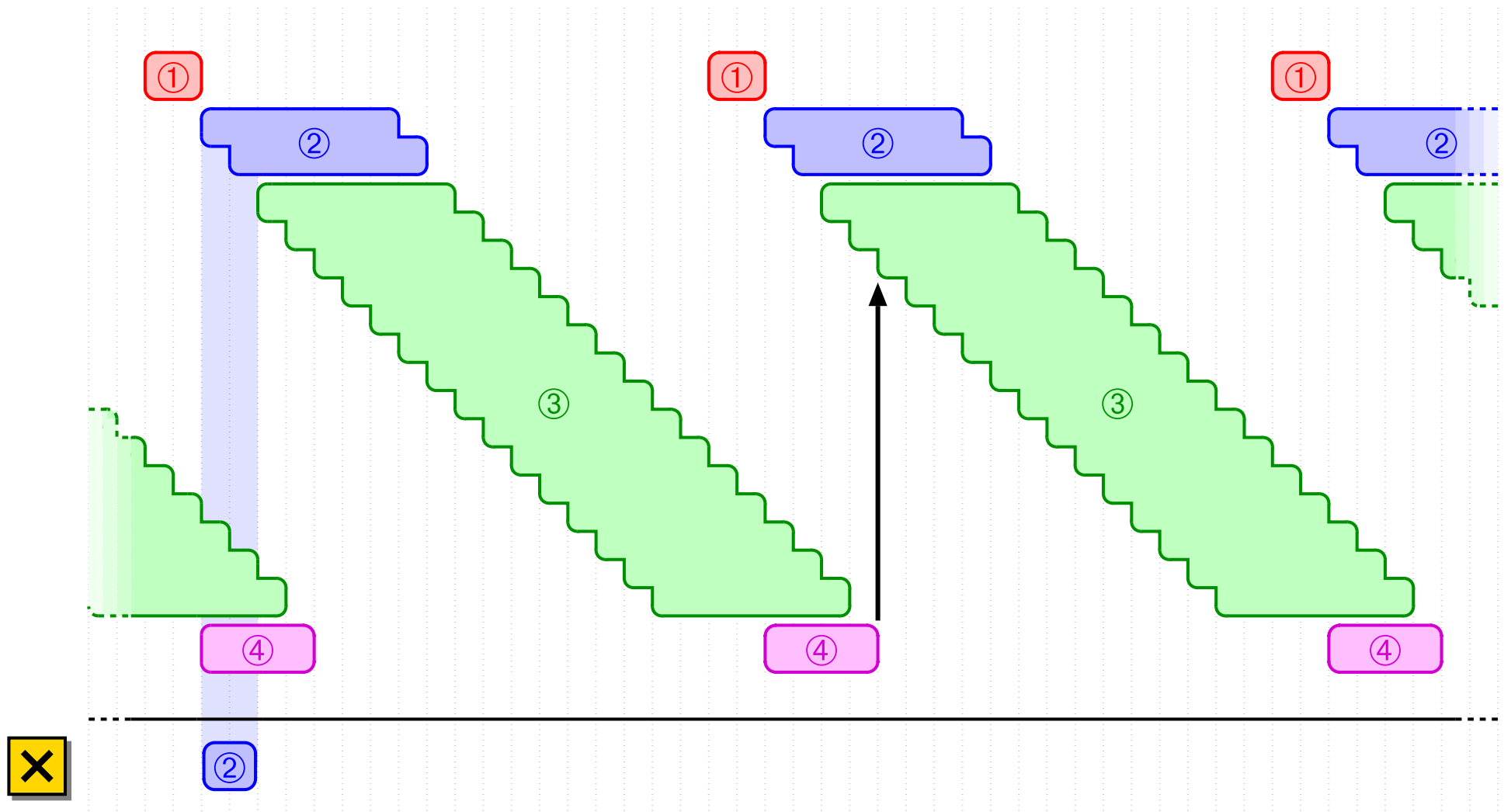
Task scheduling



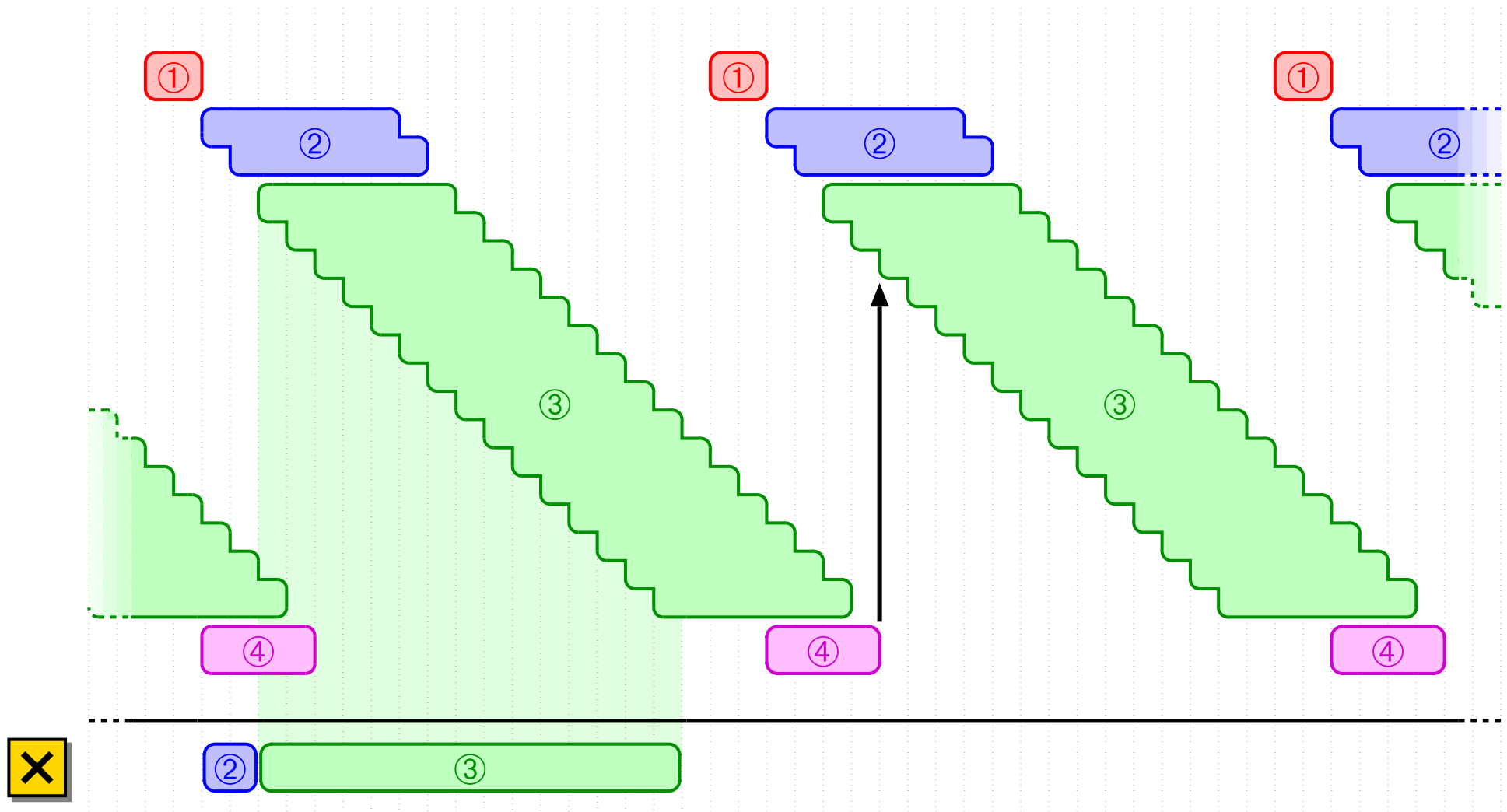
Task scheduling



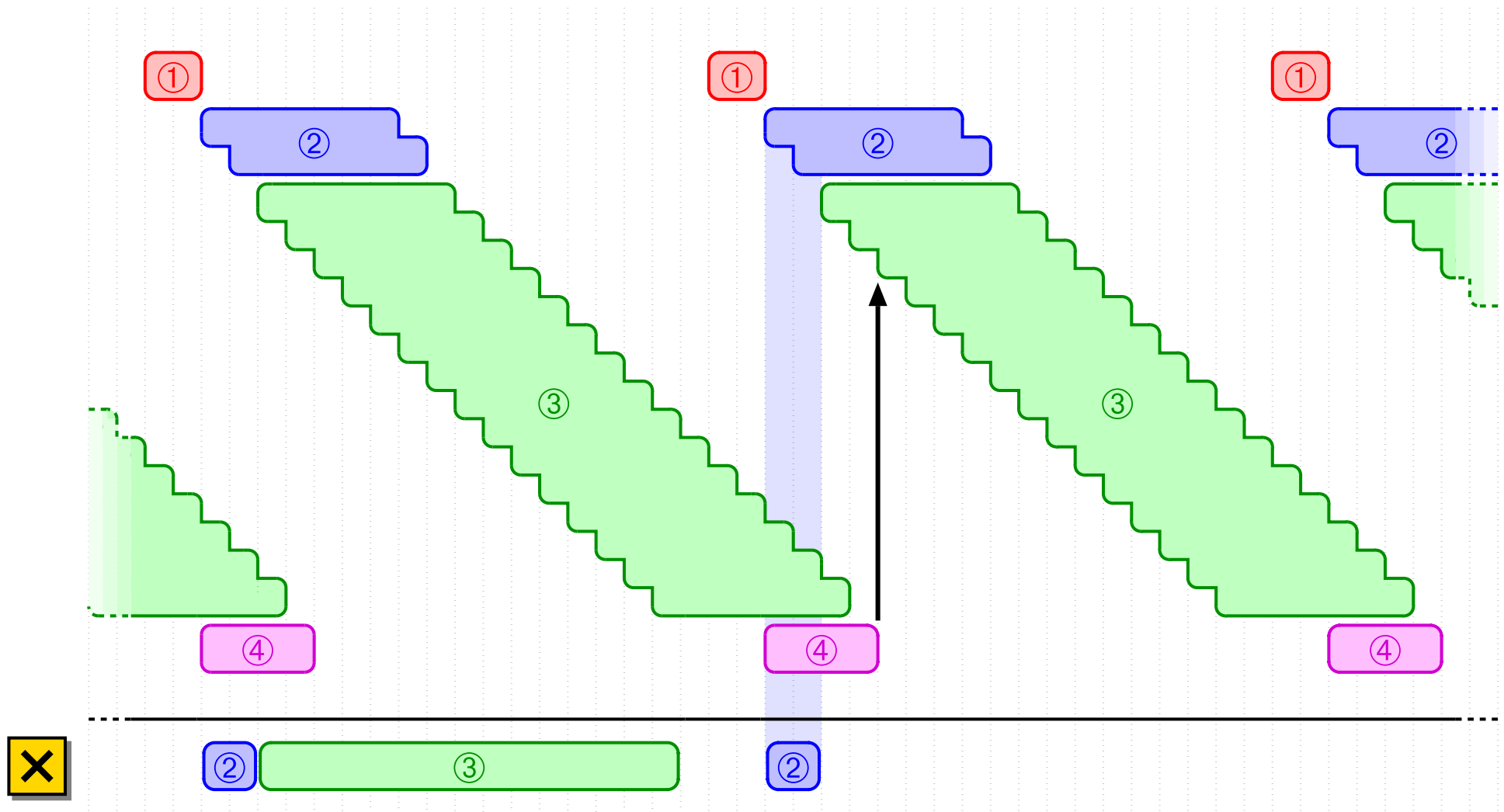
Task scheduling



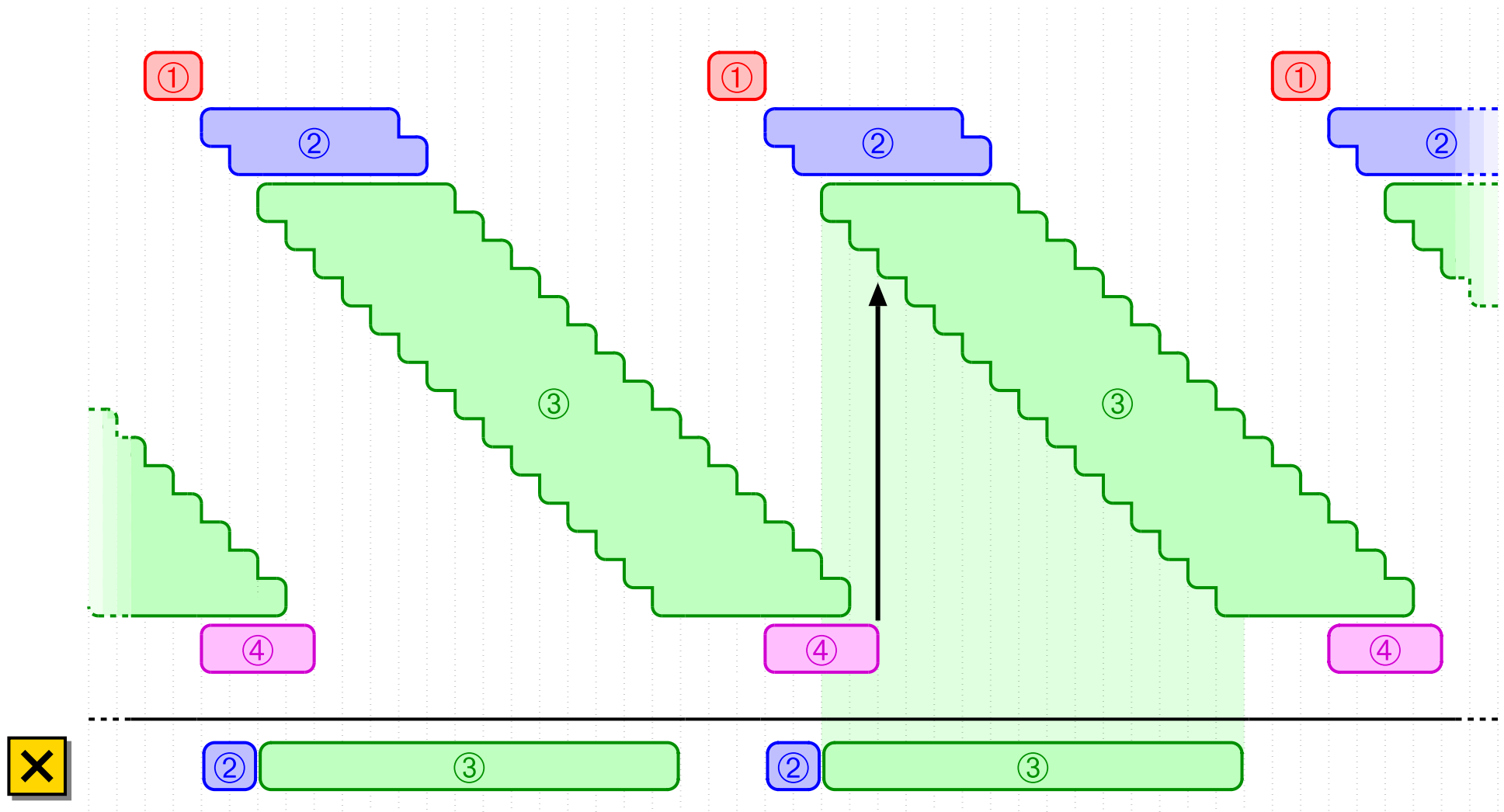
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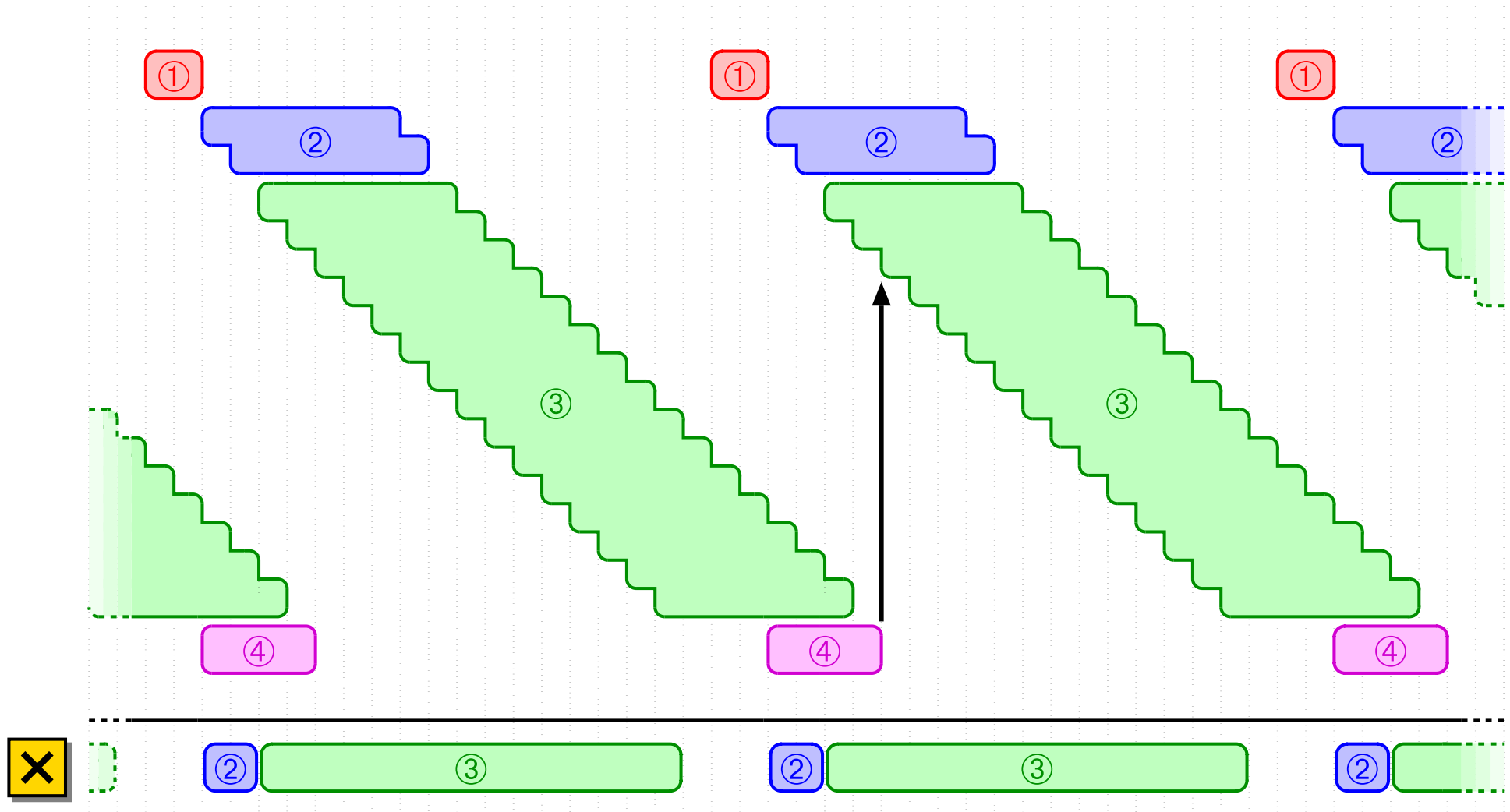
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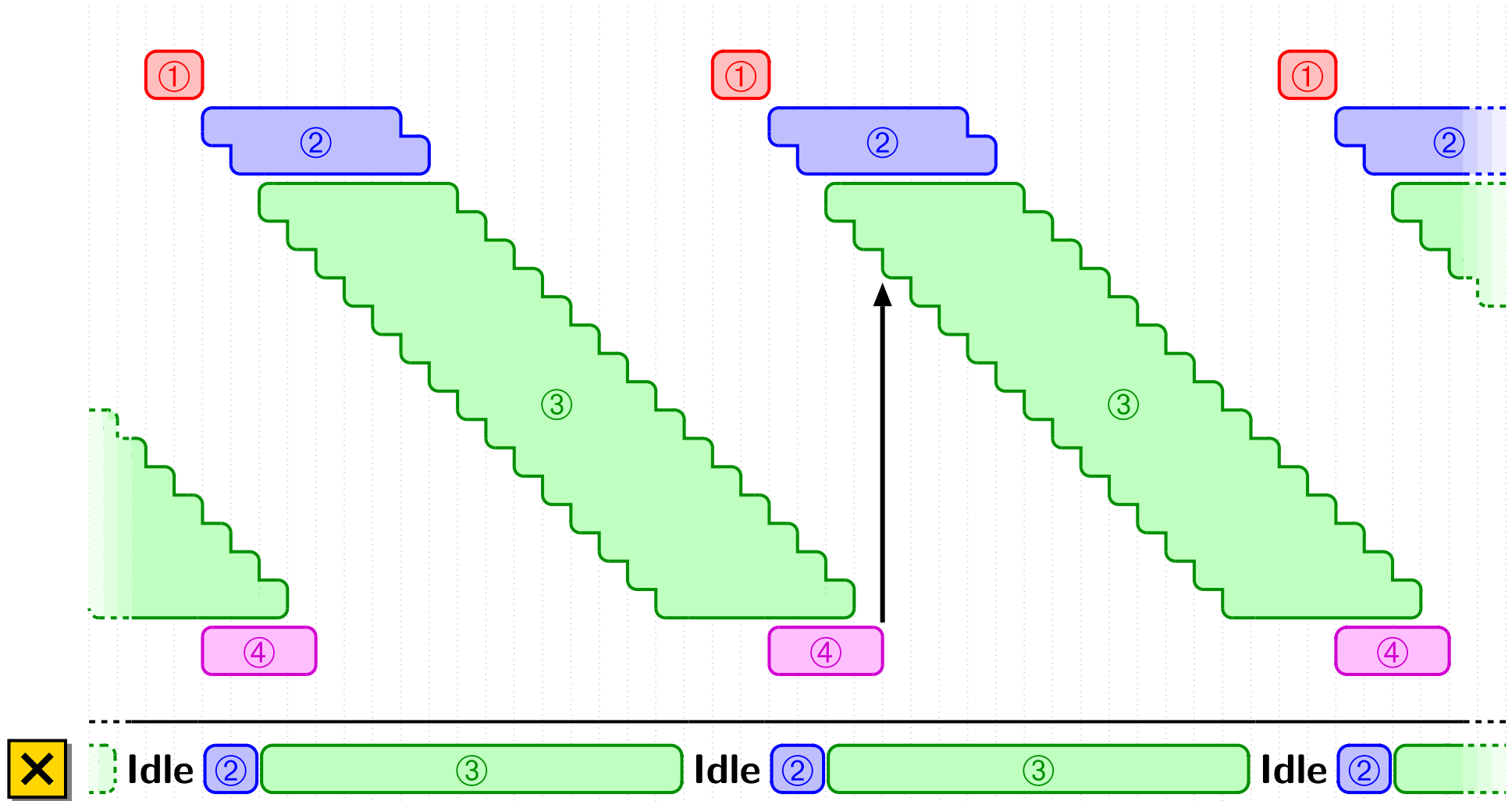
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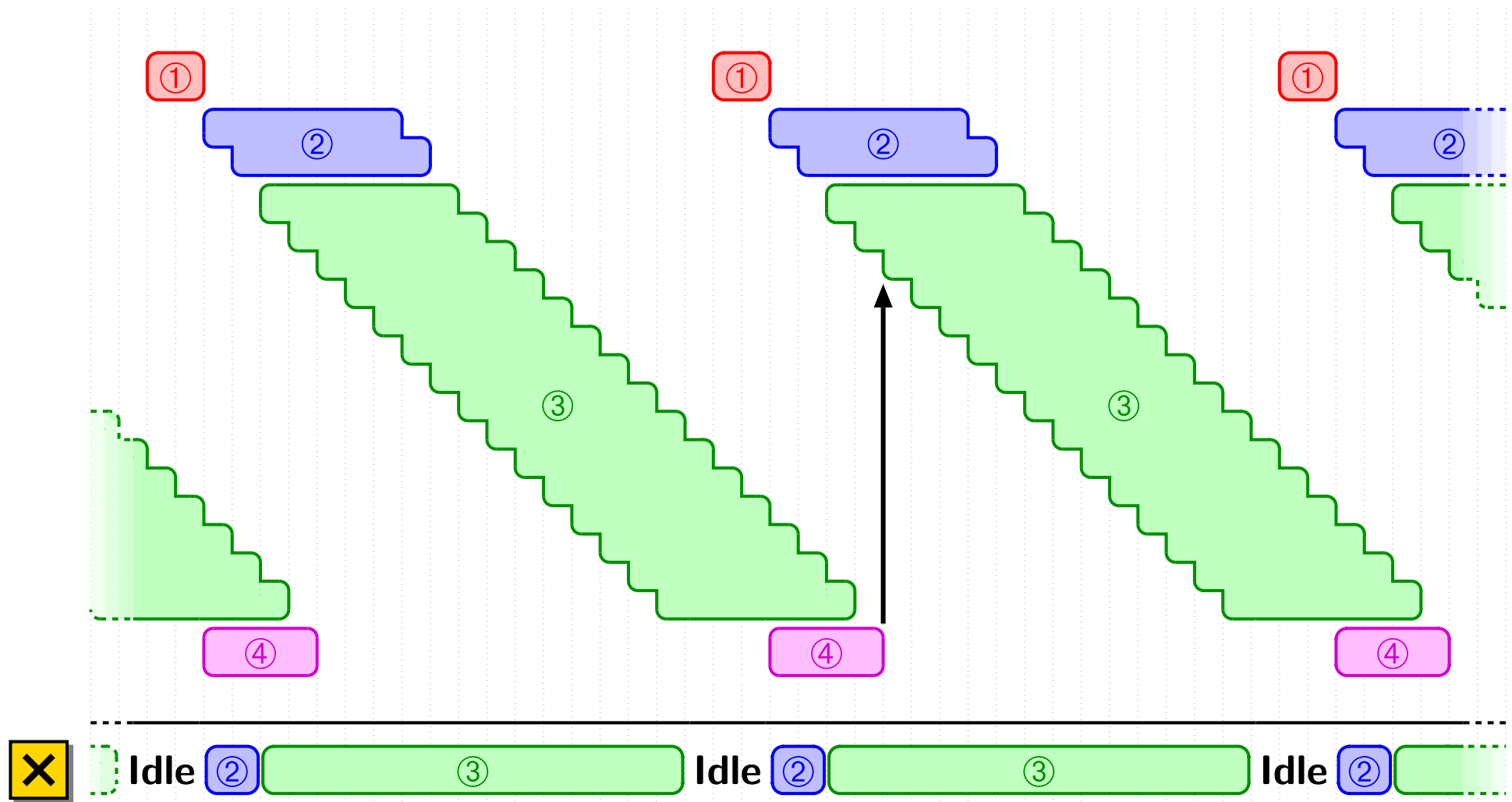
Task scheduling



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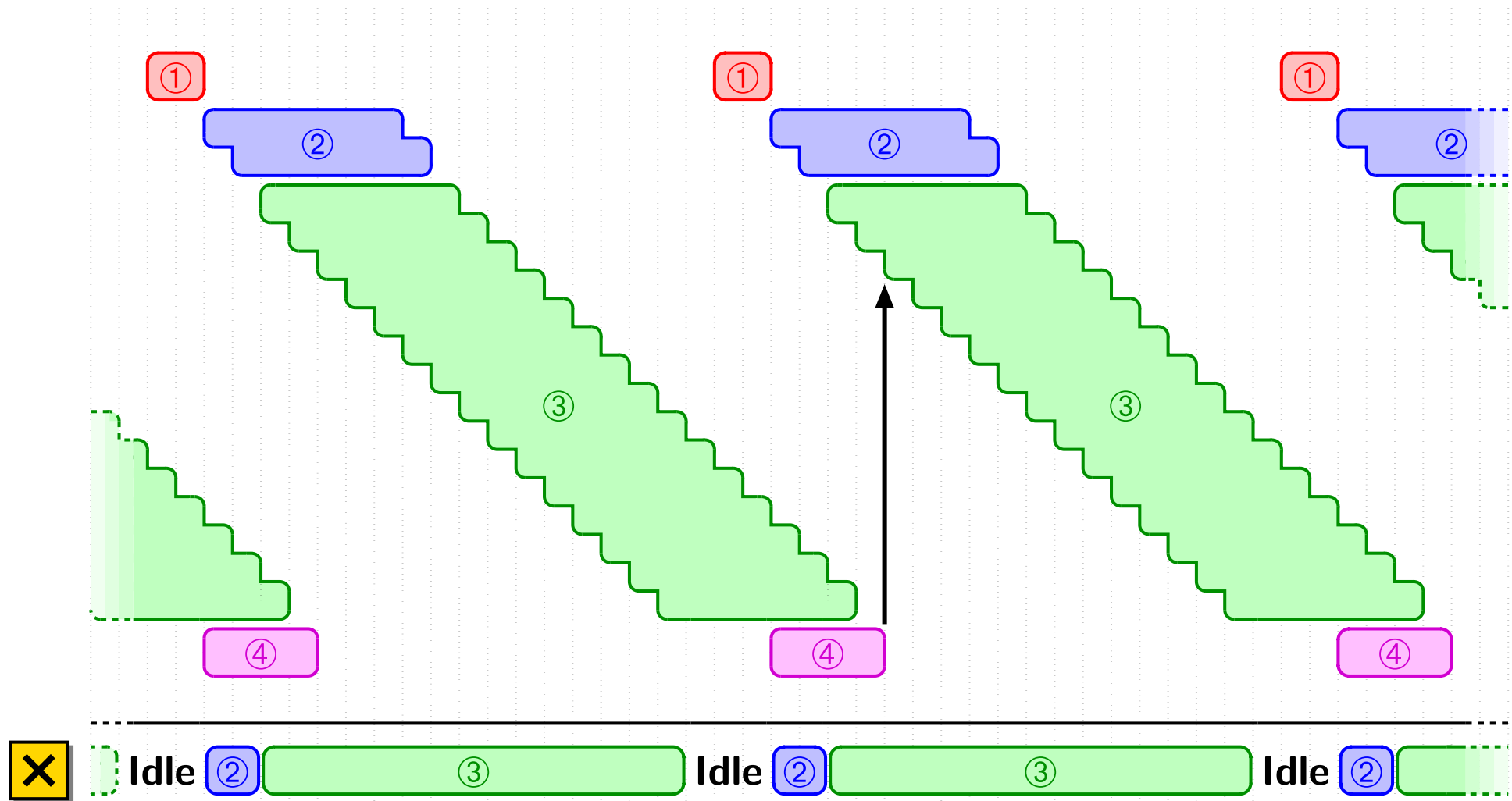


Task scheduling



► The task dependency ④ → ③ forces idle cycles into the multiplier pipeline

Task scheduling



- ▶ The task dependency ④ → ③ forces idle cycles into the multiplier pipeline
- ▶ The considered η_T pairing algorithm is not suited for parallel implementation

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

① $x_Q \leftarrow x_Q^9 \pm 1 ; y_Q \leftarrow -y_Q^9$

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end for

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end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{36m}^\times$$

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end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

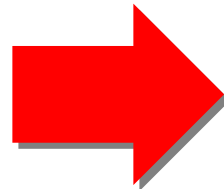
① $x_P \leftarrow x_P \quad ; \quad y_P \leftarrow y_P$
 $x_Q \leftarrow x_Q^9 \pm 1 \quad ; \quad y_Q \leftarrow -y_Q^9$

② $t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$
 $S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

④ $R \leftarrow R^3$

end for



for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

① $x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P}$
 $x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3$

② $t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$
 $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

④ $R \leftarrow R$

end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ to $(m - 1)/2$ do

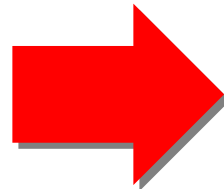
$$\textcircled{1} \quad \begin{array}{l} x_P \leftarrow x_P \quad ; \quad y_P \leftarrow y_P \\ x_Q \leftarrow x_Q^9 \pm 1 \quad ; \quad y_Q \leftarrow -y_Q^9 \end{array}$$

$$\textcircled{2} \quad \begin{array}{l} t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q \\ S \leftarrow -t^2 + u\sigma - t\rho - \rho^2 \end{array}$$

$$\textcircled{3} \quad R \leftarrow R \cdot S$$

$$\textcircled{4} \quad R \leftarrow R^3$$

end for



for $i \leftarrow 0$ to $(m - 1)/2$ do

$$\textcircled{1} \quad \begin{array}{l} x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P} \\ x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3 \end{array}$$

$$\textcircled{2} \quad \begin{array}{l} t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q \\ S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2 \end{array}$$

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end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

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2 inv. Frobenius
2 Frobenius (\mathbb{F}_{3^m})

② $t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$
 $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

① $x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P}$ 2 inv. Frobenius (\mathbb{F}_{3^m})
 $x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3$ 2 Frobenius

② $t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$ 2 \times , 1 + (\mathbb{F}_{3^m})
 $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

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② $t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$ $2 \times, 1 +$ (\mathbb{F}_{3^m})
 $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$ $1 \times$ $(\mathbb{F}_{3^{6m}})$

end for

A modified algorithm

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 $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$ $15 \times, 29 +$ (\mathbb{F}_{3^m})

end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

① $x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P}$ 2 inv. Frobenius
 $x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3$ 2 Frobenius (\mathbb{F}_{3^m})

② $t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$ 2 \times , 1 +
 $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$ (\mathbb{F}_{3^m})

③ $R \leftarrow R \cdot S$ 15 \times , 29 + (\mathbb{F}_{3^m})

end for

► Modified algorithm: 17 \times , 2 Frobenius, 2 inverse Frobenius and 30 + over \mathbb{F}_{3^m}

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

①	$x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P}$ $x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3$	2 inv. Frobenius 2 Frobenius	(\mathbb{F}_{3^m})
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②	$t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$ $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$	2 \times , 1 +	(\mathbb{F}_{3^m})
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③	$R \leftarrow R \cdot S$	15 \times , 29 +	(\mathbb{F}_{3^m})
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end for

► Modified algorithm: 17 \times , 2 Frobenius, 2 inverse Frobenius and 30 + over \mathbb{F}_{3^m}

► Previous algorithm: 17 \times , 10 Frobenius and 38 +

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

①	$x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P}$ $x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3$	2 inv. Frobenius 2 Frobenius	(\mathbb{F}_{3^m})
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②	$t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$ $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$	2 \times , 1 +	(\mathbb{F}_{3^m})
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③	$R \leftarrow R \cdot S$	15 \times , 29 +	(\mathbb{F}_{3^m})
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end for

- ▶ Modified algorithm: 17 \times , 2 Frobenius, 2 inverse Frobenius and 30 + over \mathbb{F}_{3^m}
- ▶ Previous algorithm: 17 \times , 10 Frobenius and 38 +
- ▶ Cost of the inverse Frobenius?

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m - 1)/2$ **do**

①	$x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P}$ $x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3$	2 inv. Frobenius 2 Frobenius	(\mathbb{F}_{3^m})
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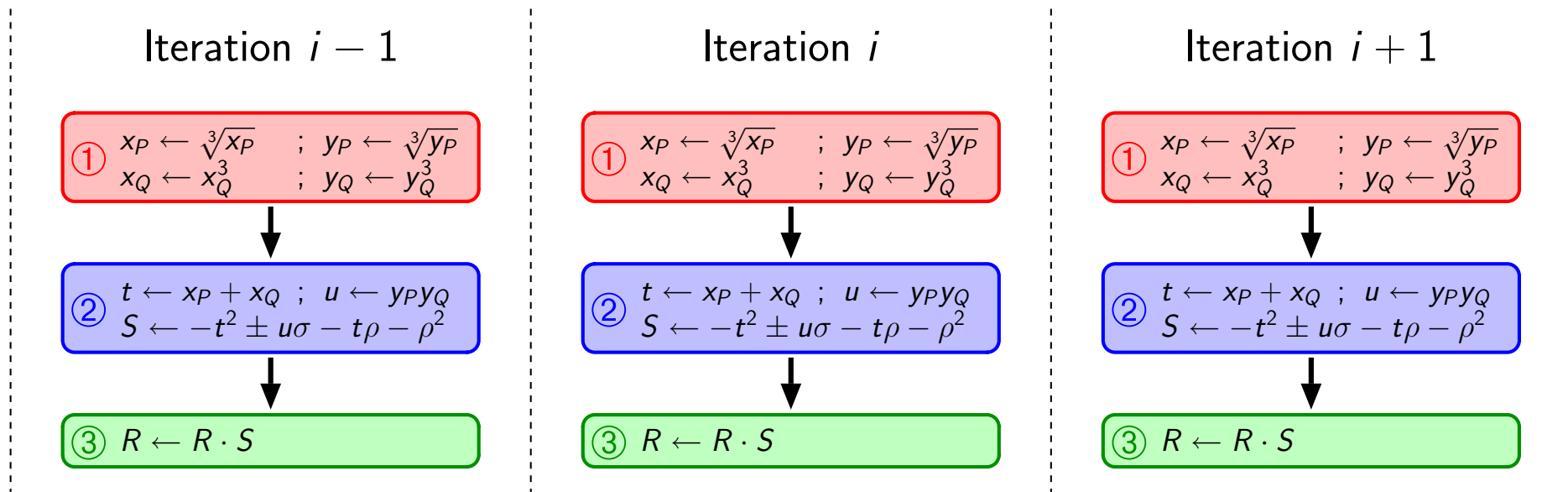
②	$t \leftarrow x_P + x_Q \quad ; \quad u \leftarrow y_P y_Q$ $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$	2 \times , 1 +	(\mathbb{F}_{3^m})
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③	$R \leftarrow R \cdot S$	15 \times , 29 +	(\mathbb{F}_{3^m})
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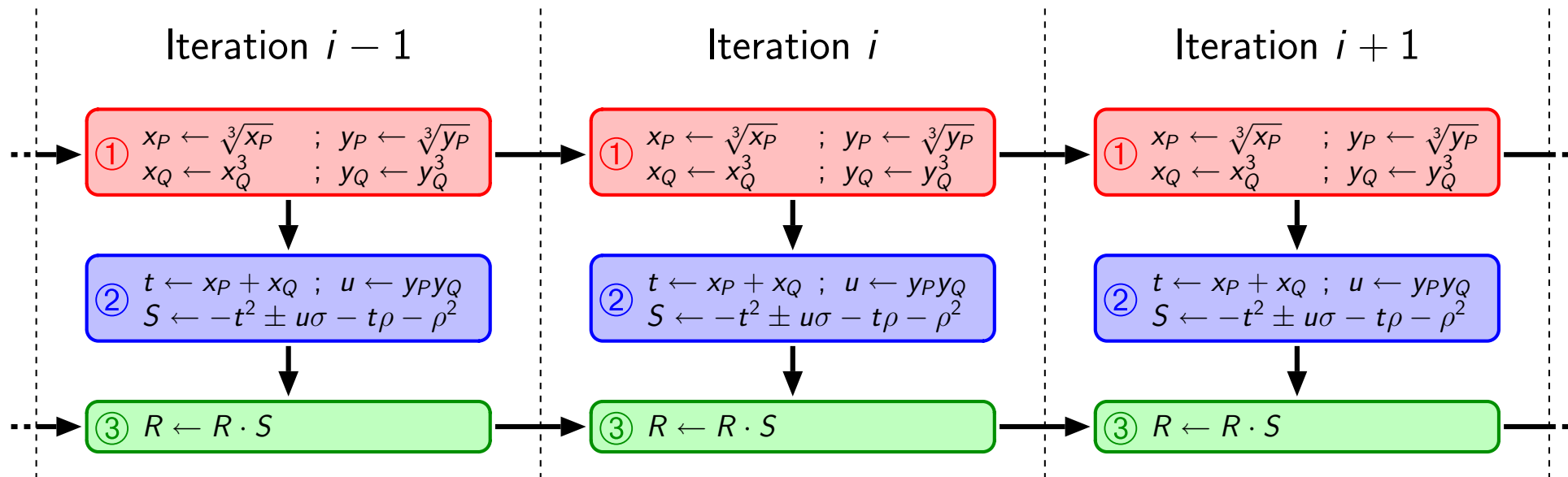
end for

- ▶ Modified algorithm: 17 \times , 2 Frobenius, 2 inverse Frobenius and 30 + over \mathbb{F}_{3^m}
- ▶ Previous algorithm: 17 \times , 10 Frobenius and 38 +
- ▶ Cost of the **inverse Frobenius**? Same as the **Frobenius**

Data dependencies

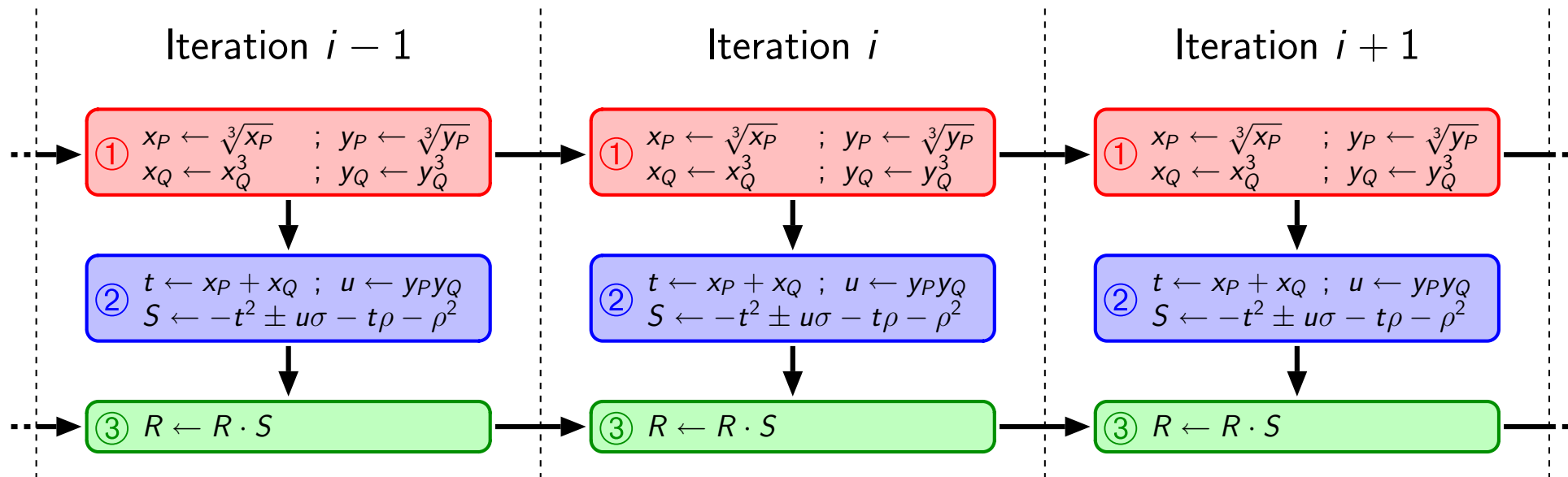


Data dependencies



► Direct dependency ③ \rightarrow ③ between consecutive iterations

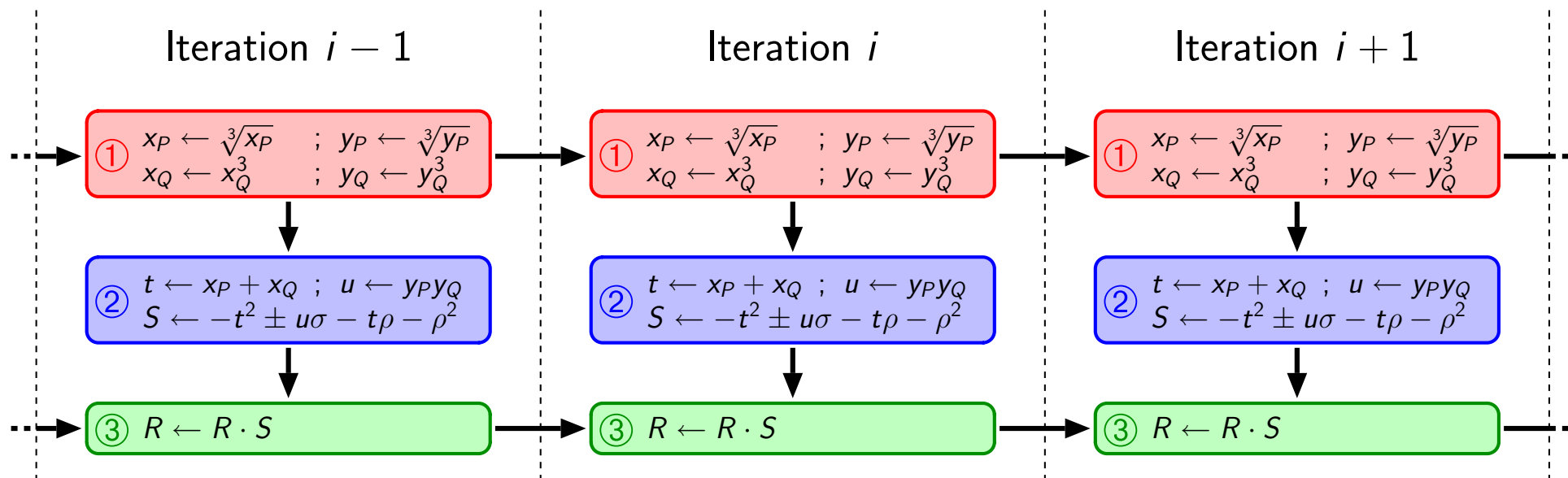
Data dependencies



► Direct dependency ③ → ③ between consecutive iterations

- avoid the scheduling bottleneck of task ④
- better overlapping of successive tasks ③

Data dependencies



► Direct dependency ③ → ③ between consecutive iterations

- avoid the scheduling bottleneck of task ④
- better overlapping of successive tasks ③
- hopefully tighter scheduling

Task scheduling



Task scheduling

$$\textcircled{1} \quad \begin{array}{l} x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P} \\ x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3 \end{array}$$

Task scheduling

① 2 inv. Frobenius
2 Frobenius

Task scheduling

①

Task scheduling

①

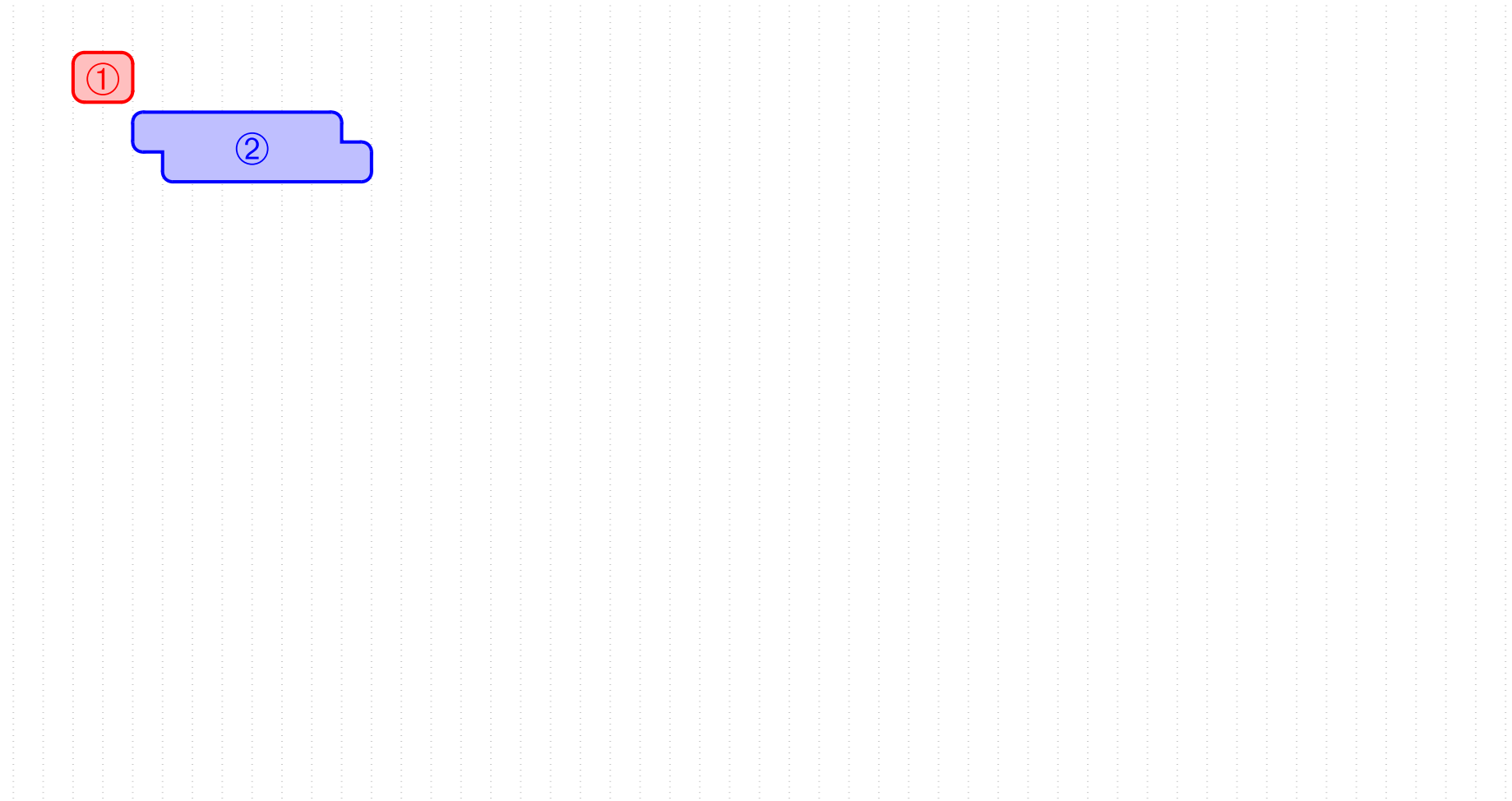
② $t \leftarrow x_P + x_Q ; u \leftarrow y_P y_Q$
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Task scheduling

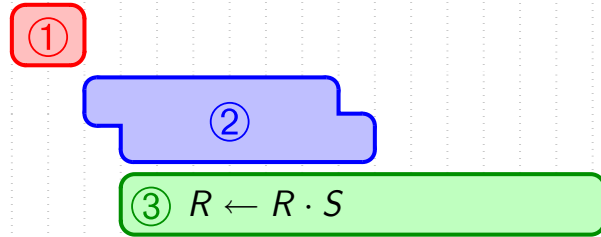
①

② $2 \times, 1 +$

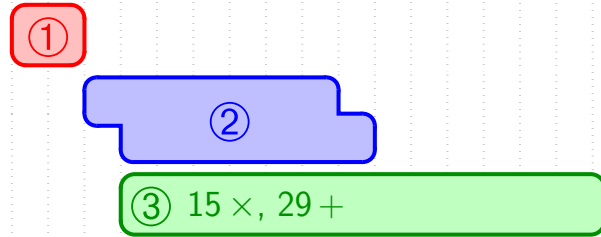
Task scheduling



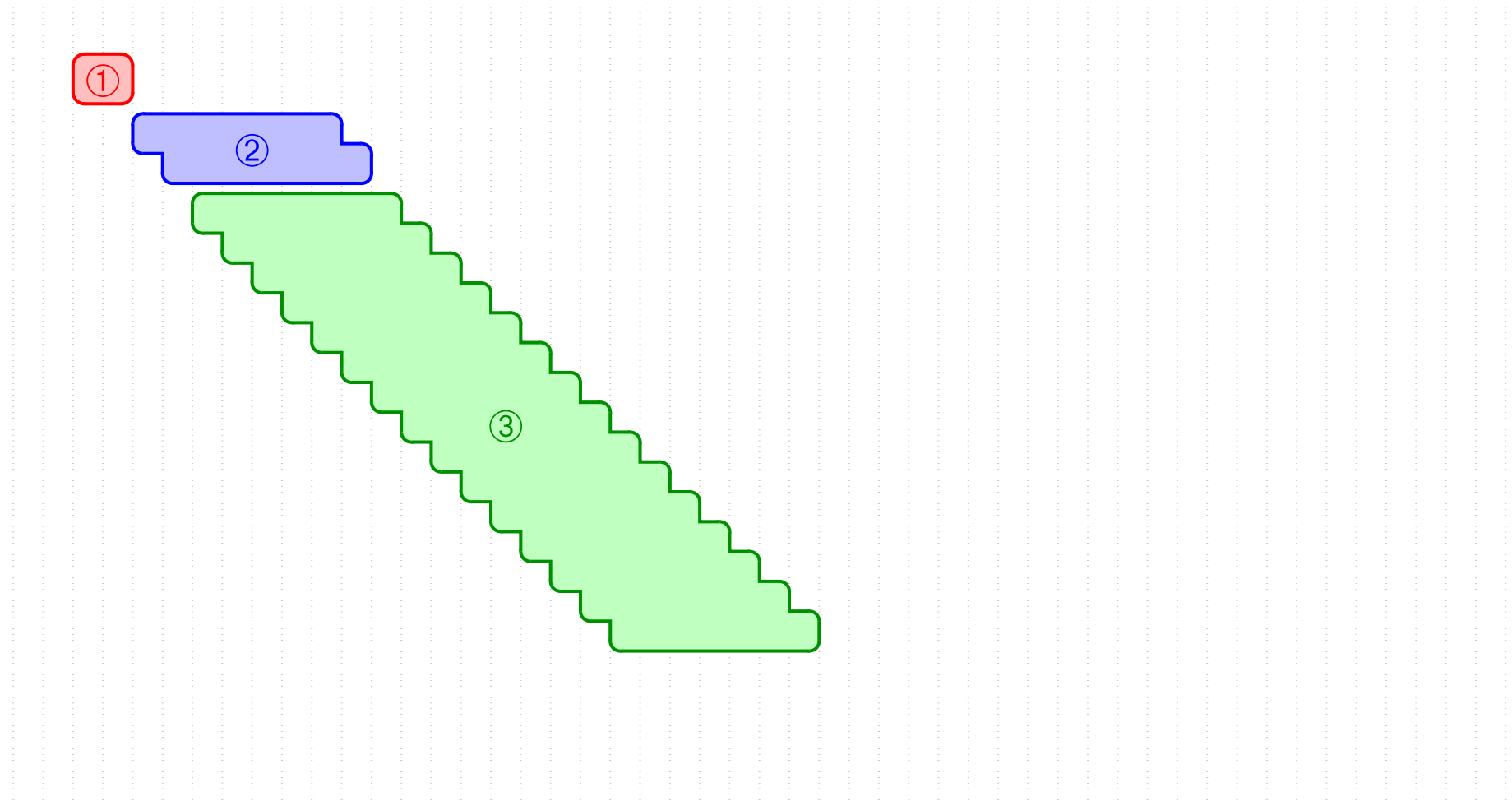
Task scheduling



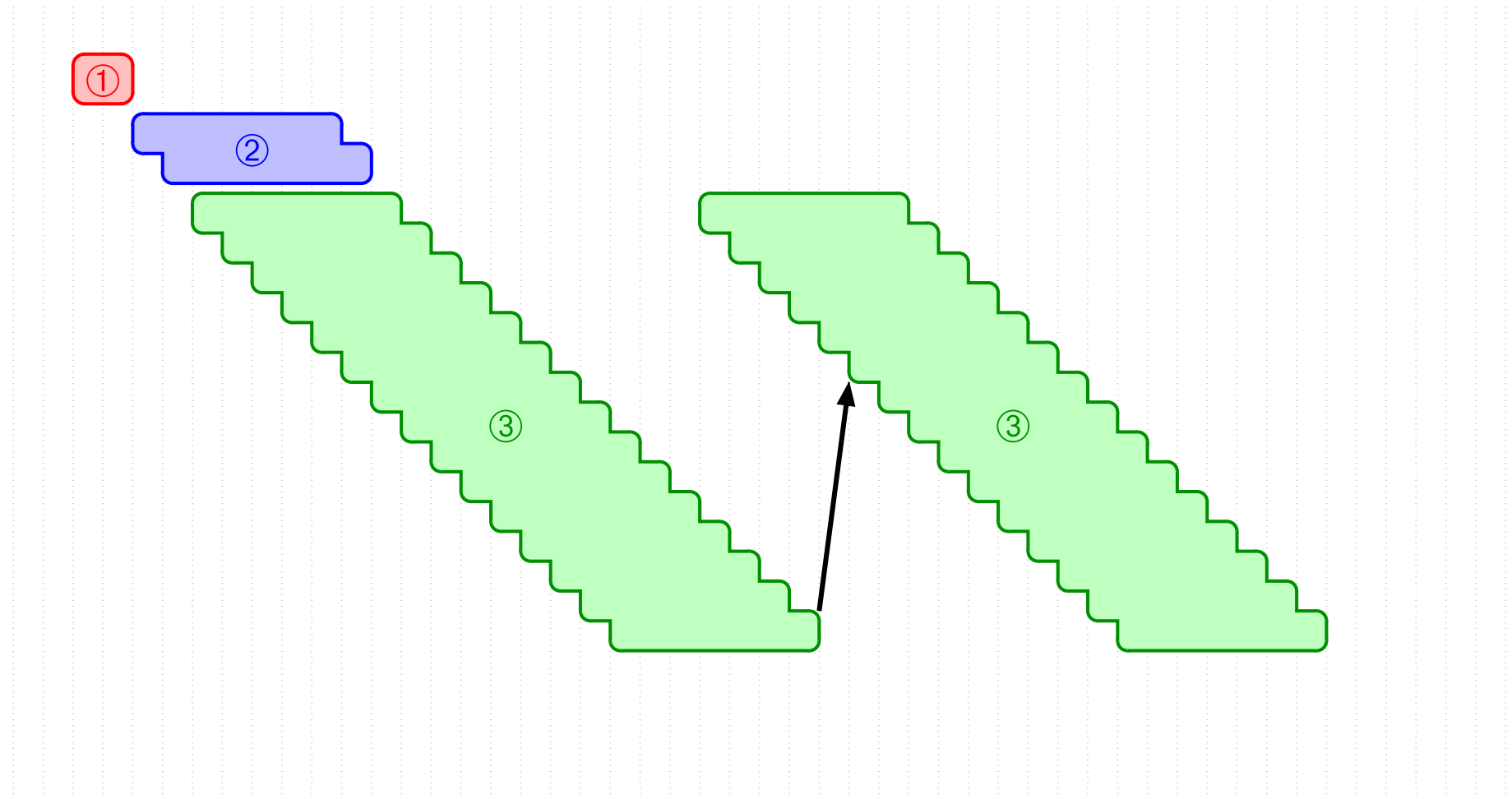
Task scheduling



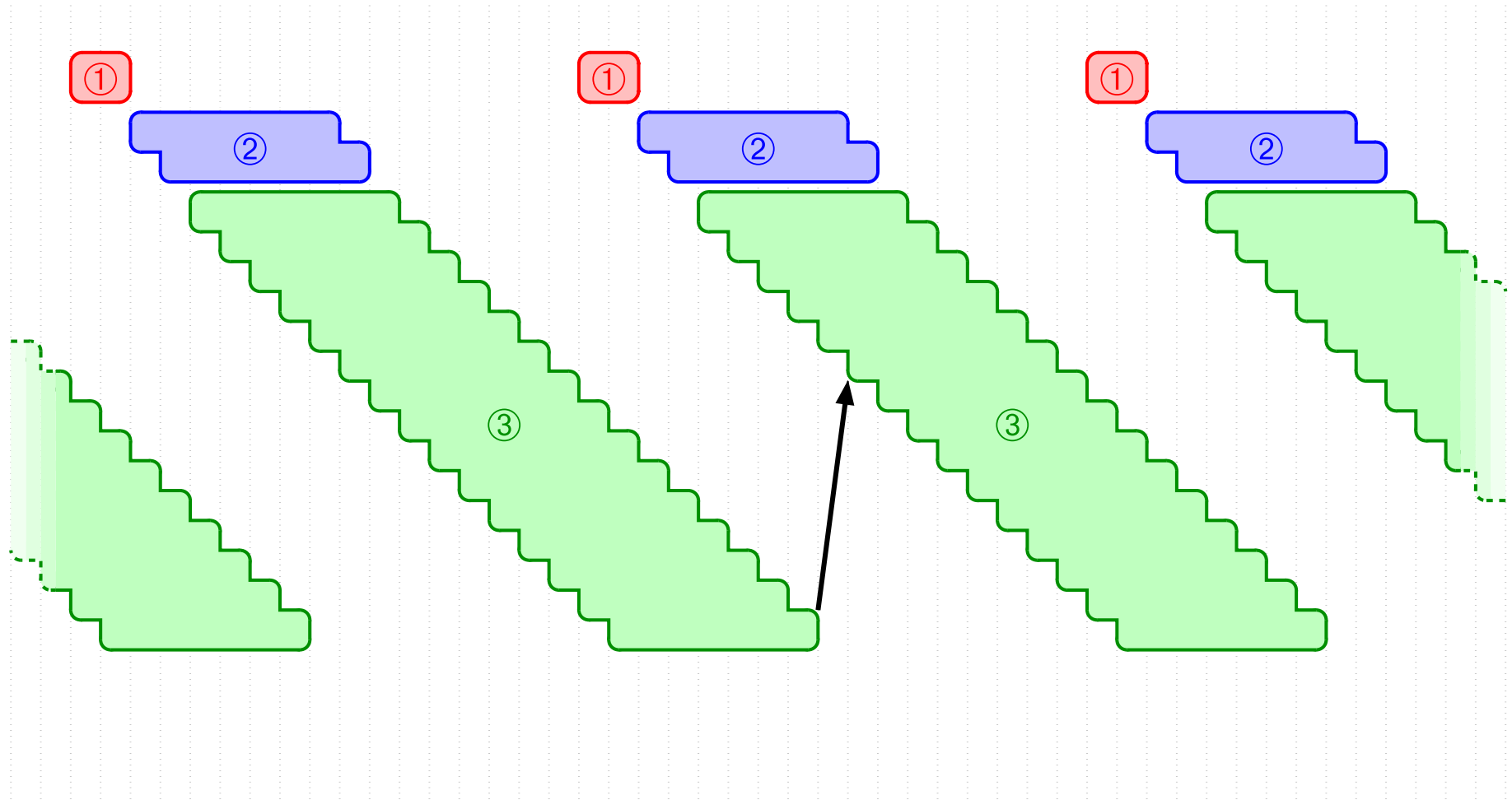
Task scheduling



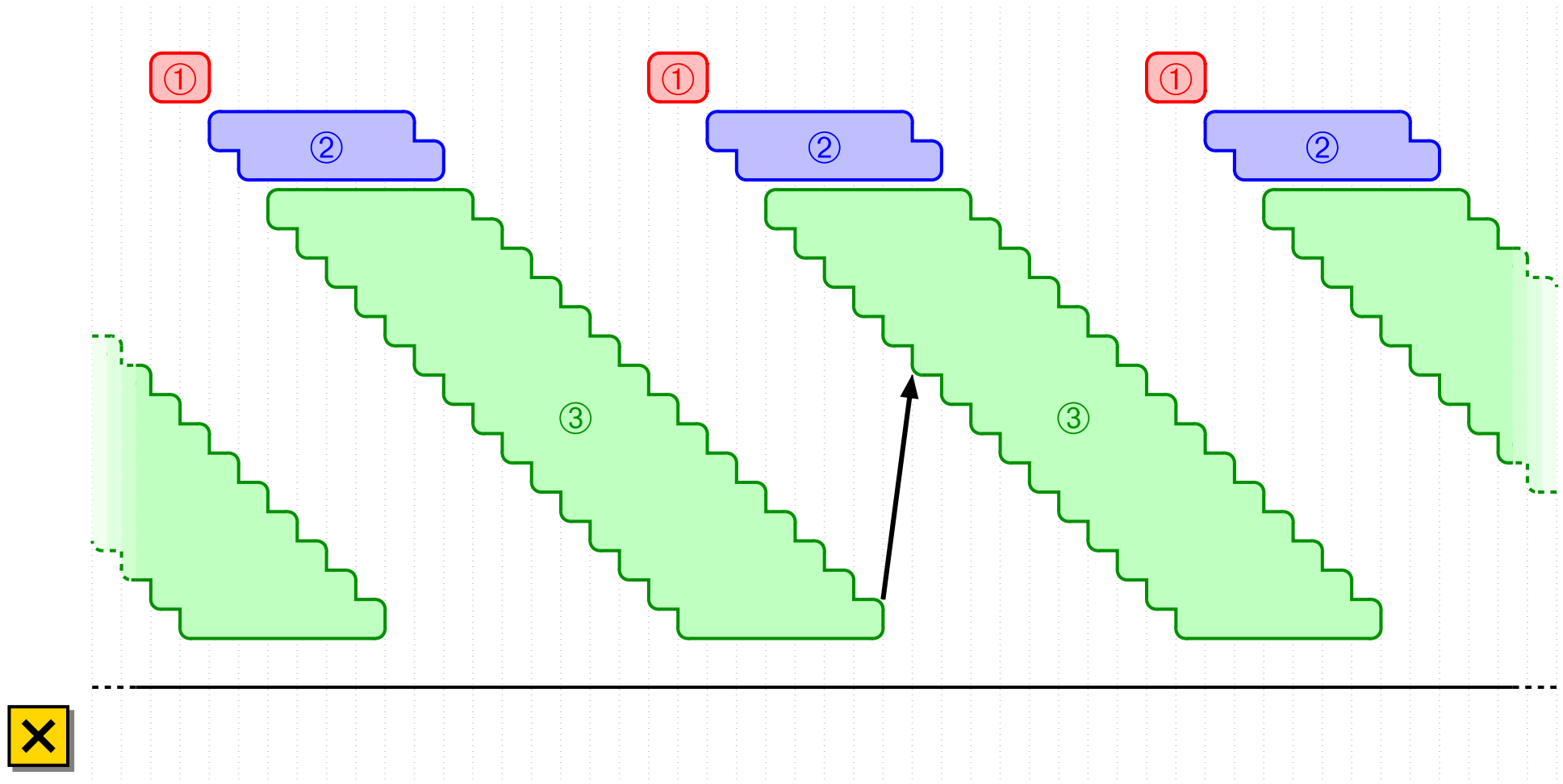
Task scheduling



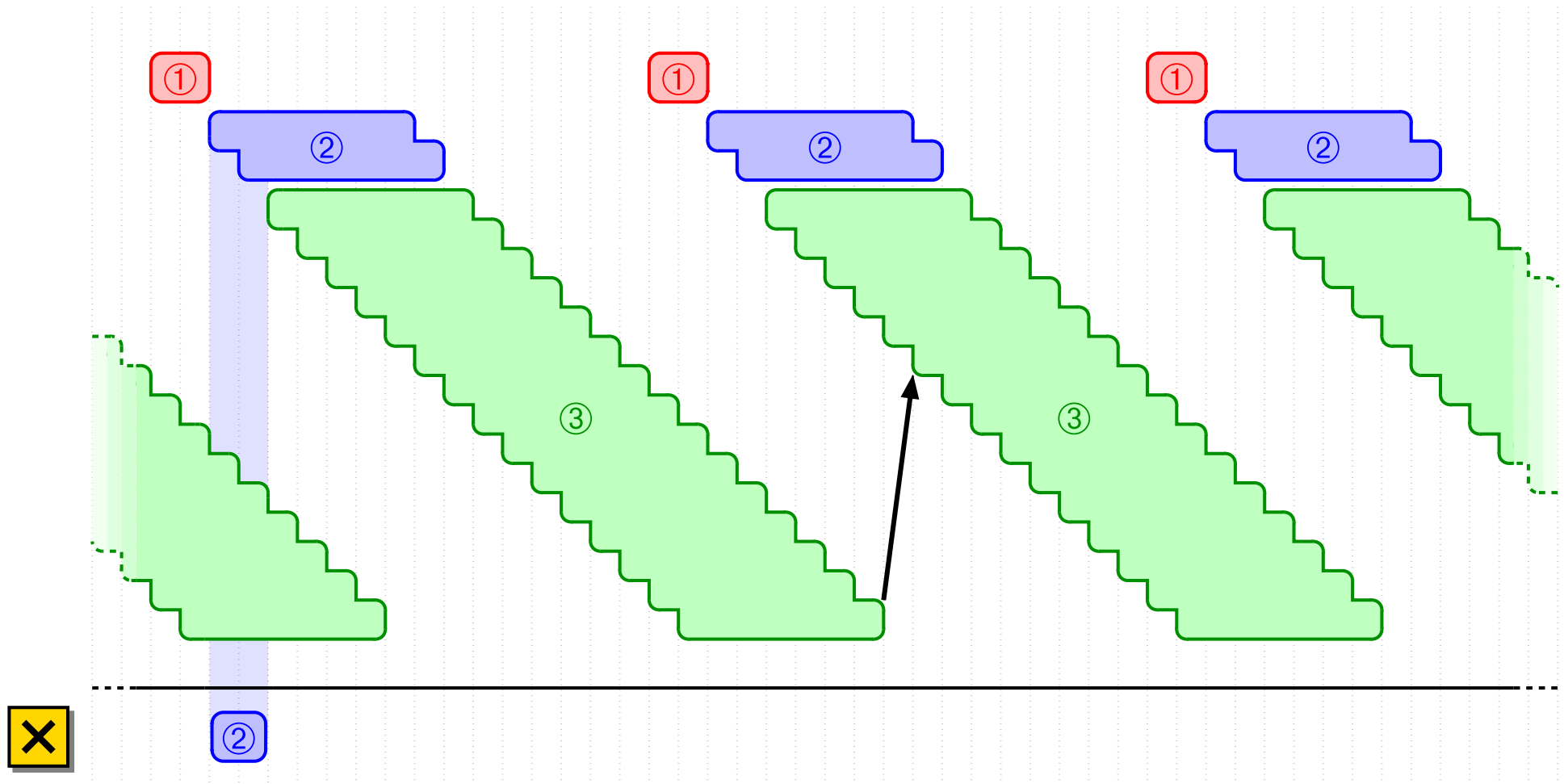
Task scheduling



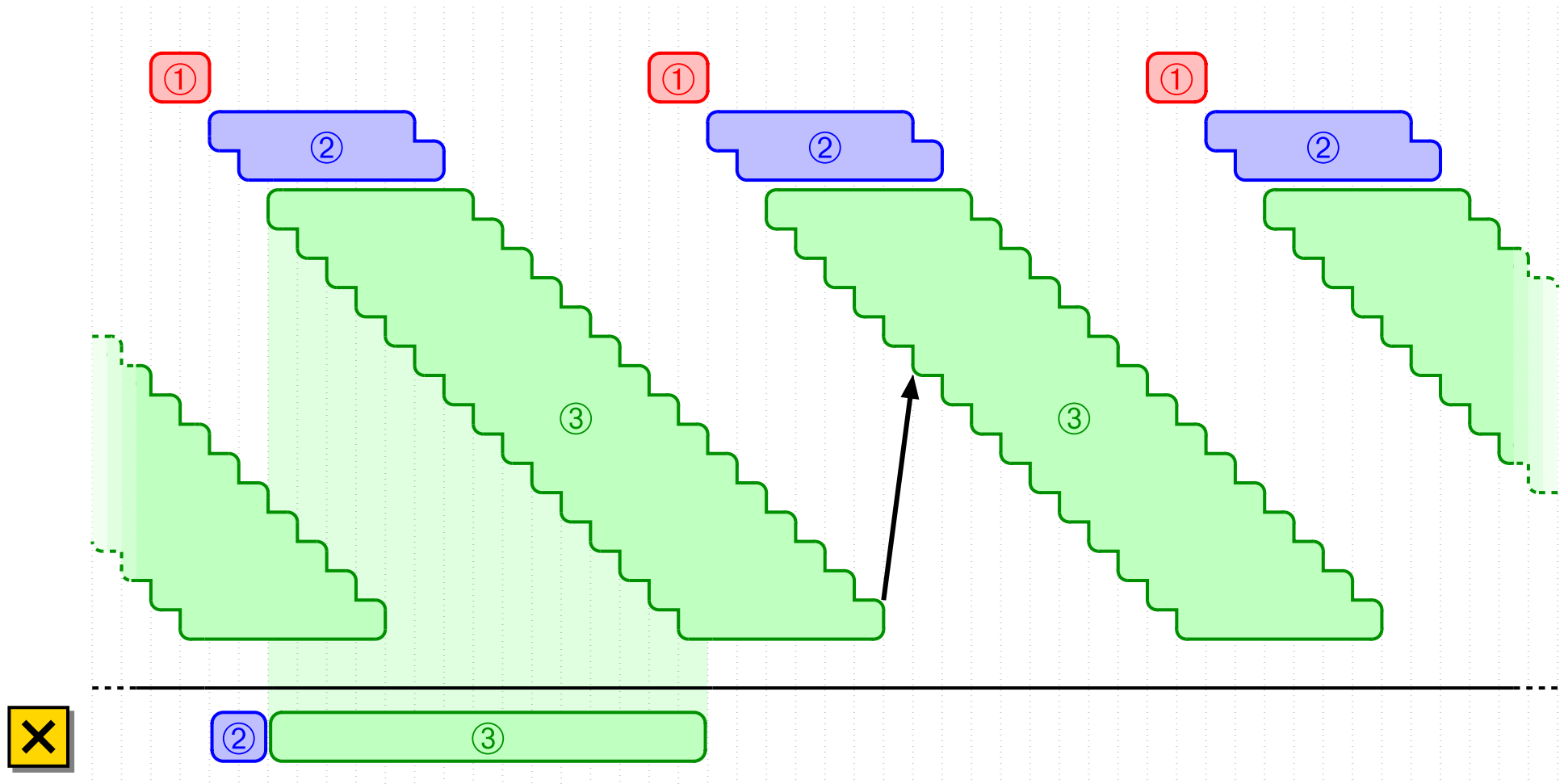
Task scheduling



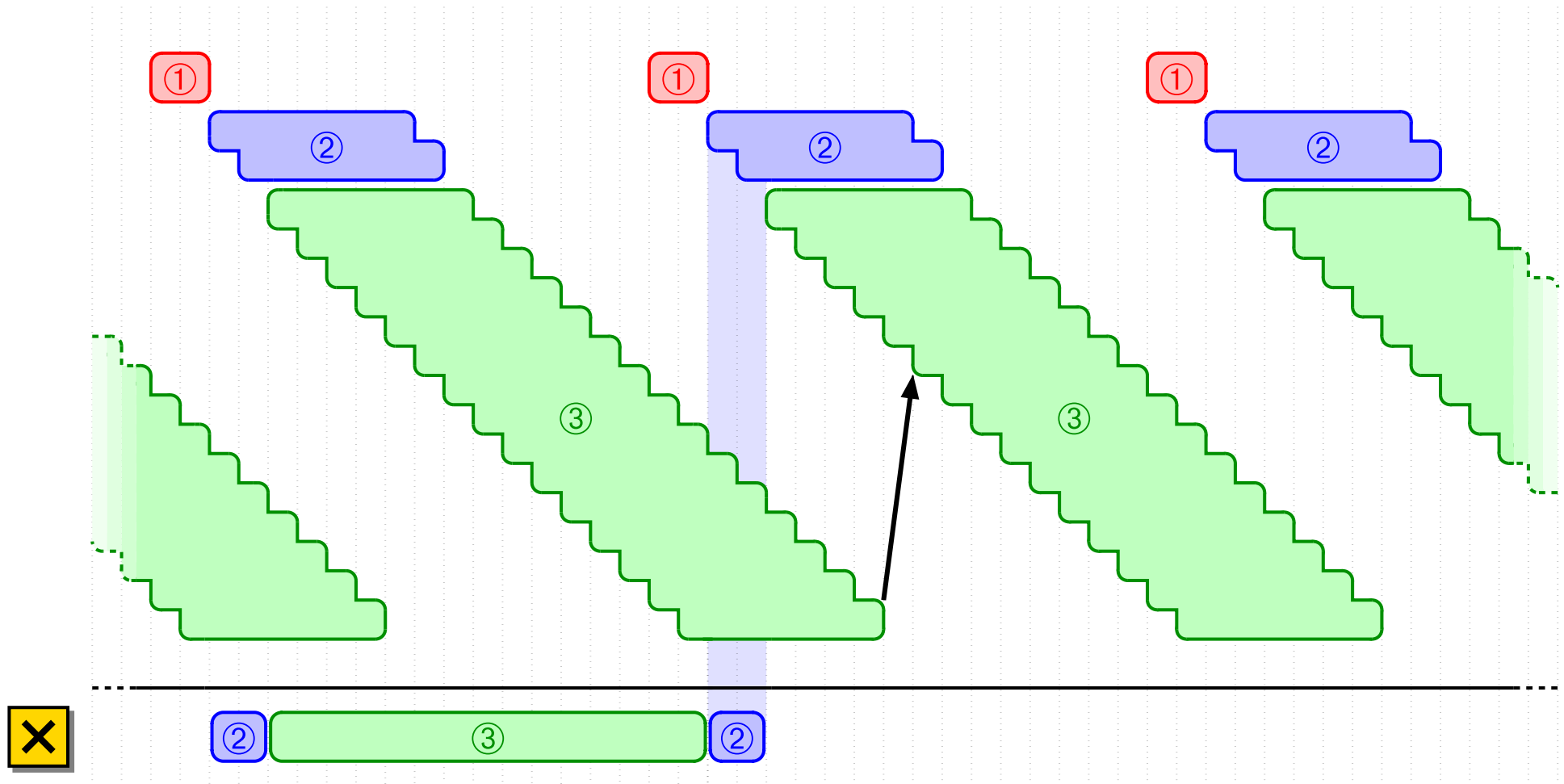
Task scheduling



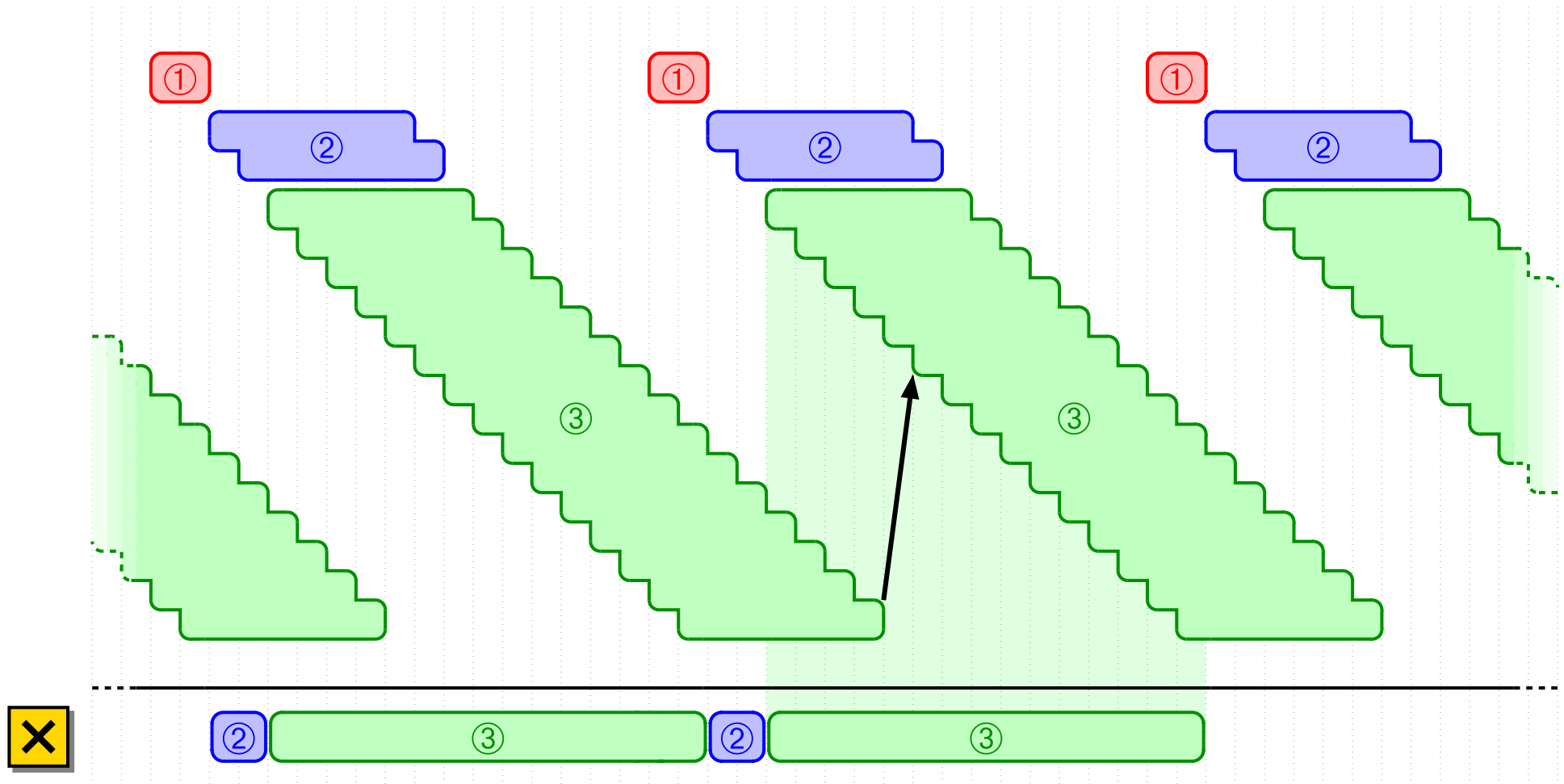
Task scheduling



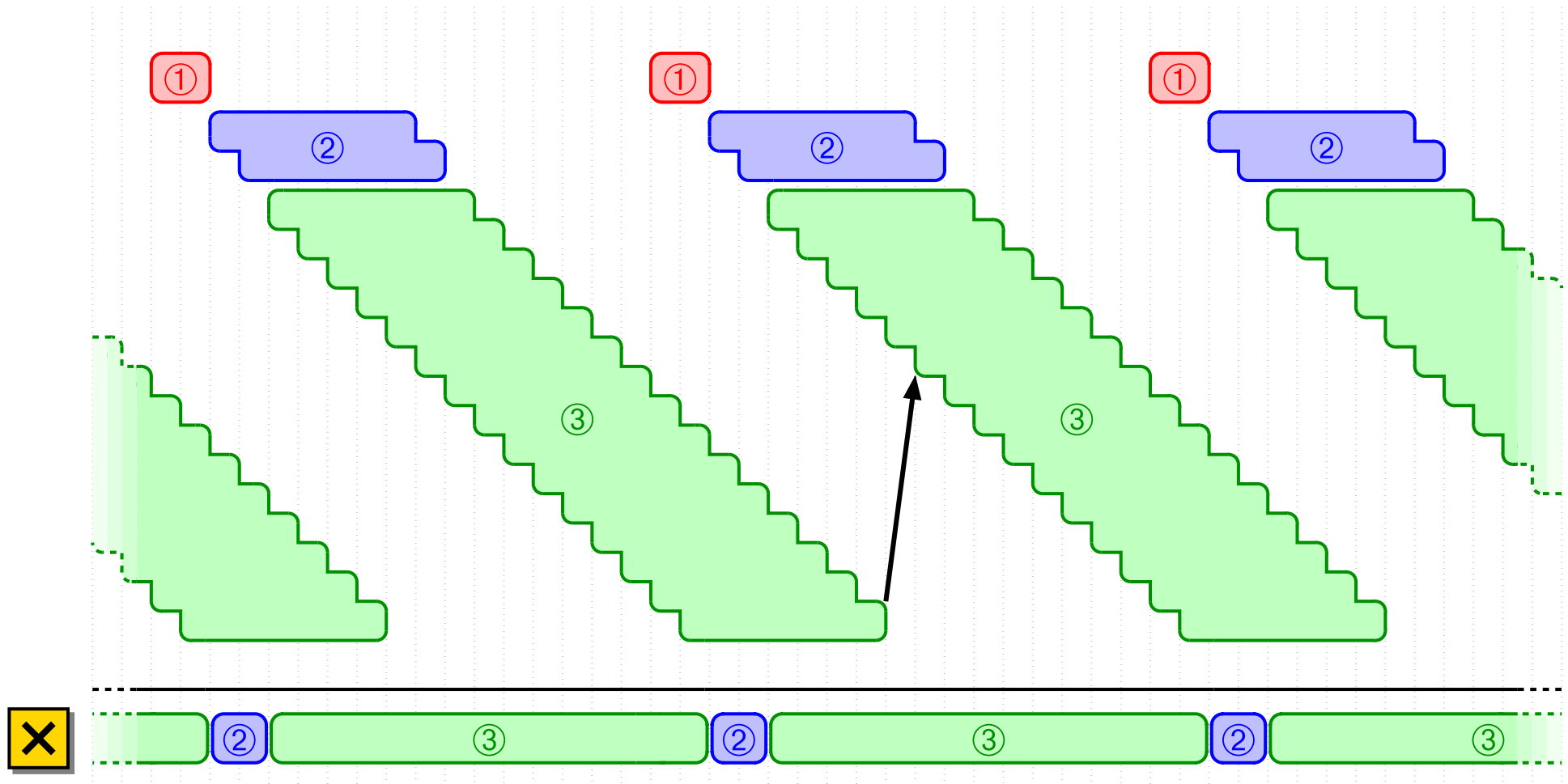
Task scheduling



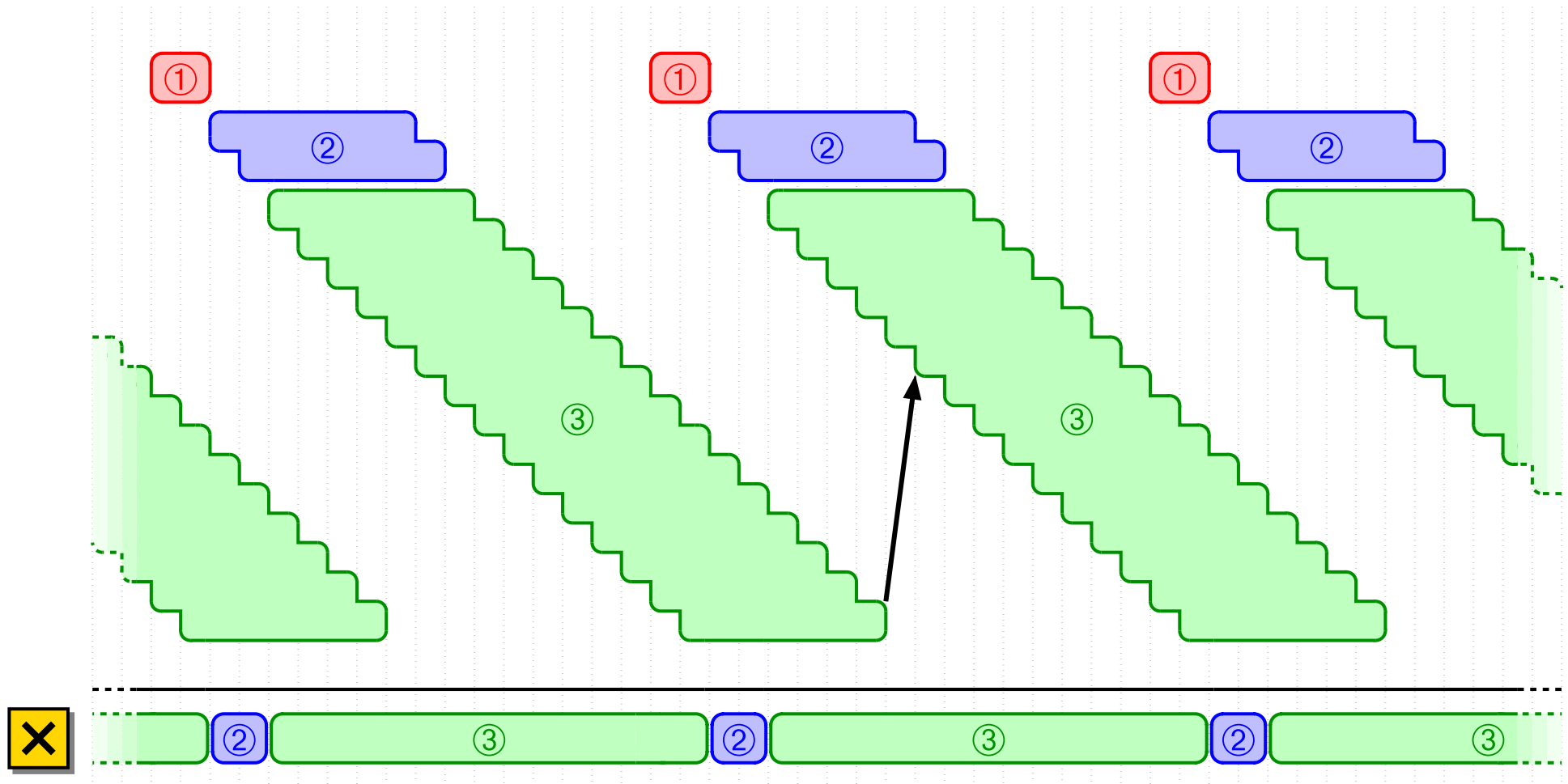
Task scheduling



Task scheduling

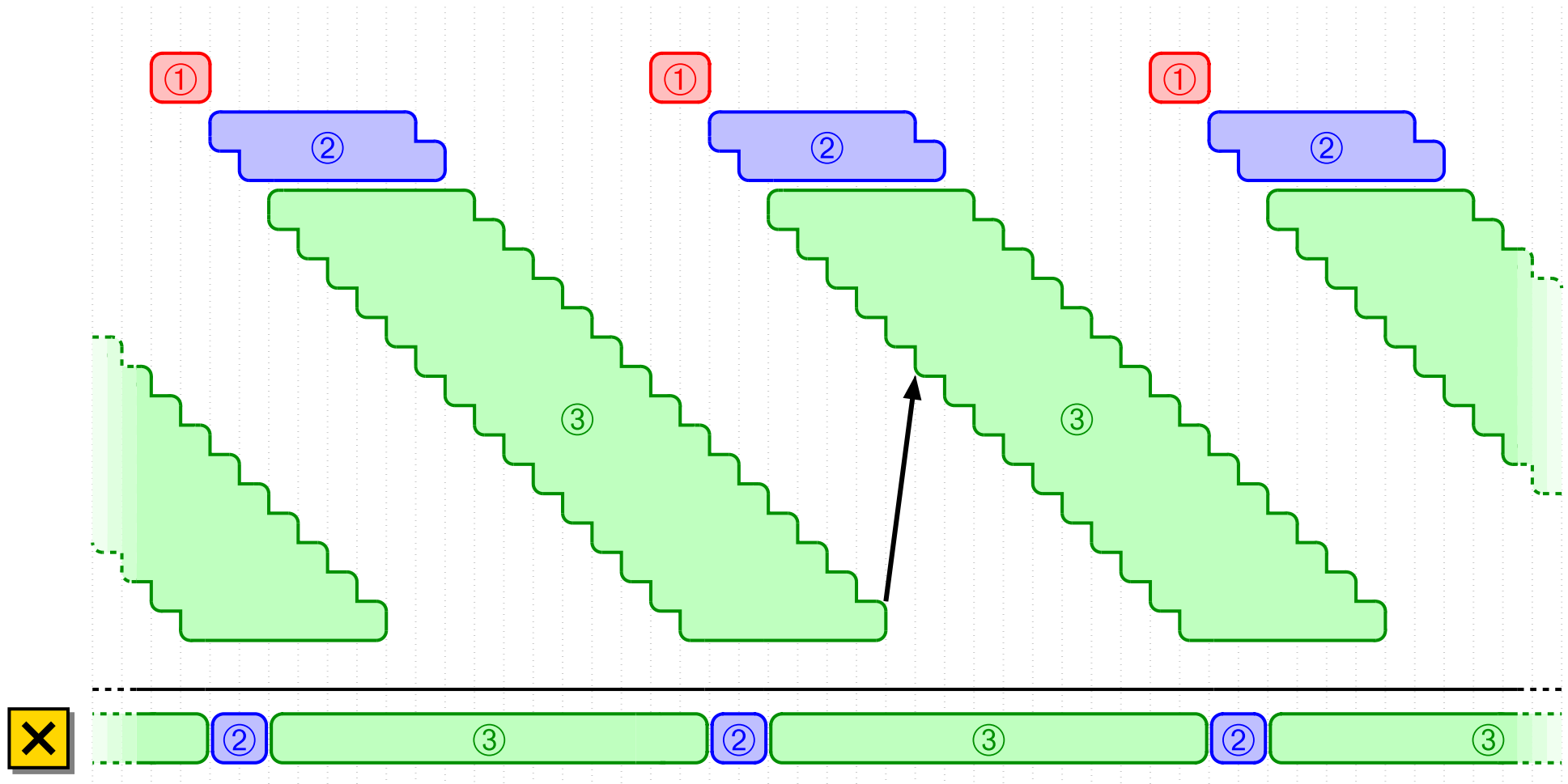


Task scheduling



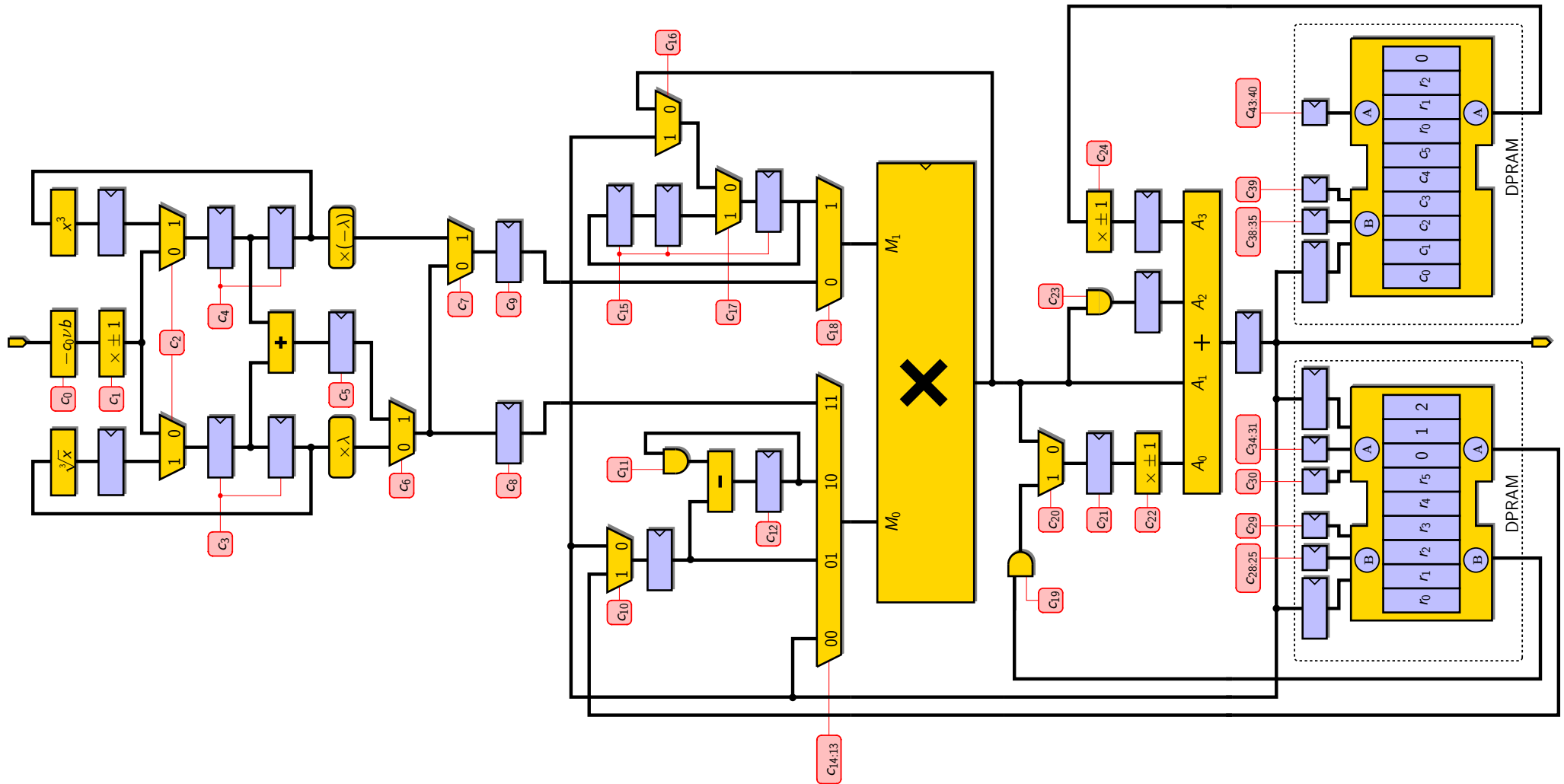
- ▶ Perfectly tight scheduling: no idle cycle
- ▶ 17 clock cycles per iteration

Task scheduling

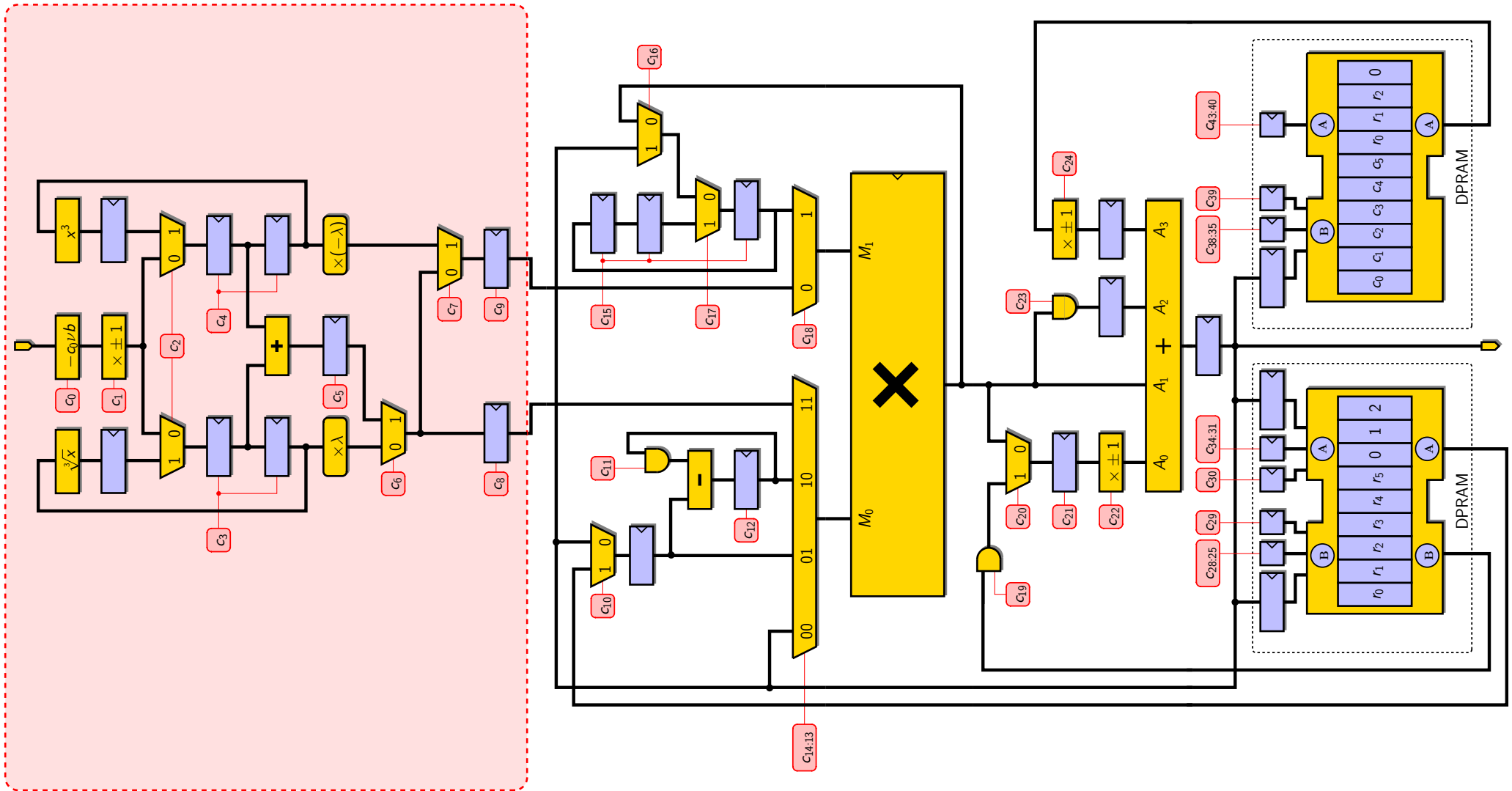


- ▶ Perfectly tight scheduling: no idle cycle
- ▶ 17 clock cycles per iteration $\Rightarrow 17(m + 1)/2$ cycles for the complete η_T pairing

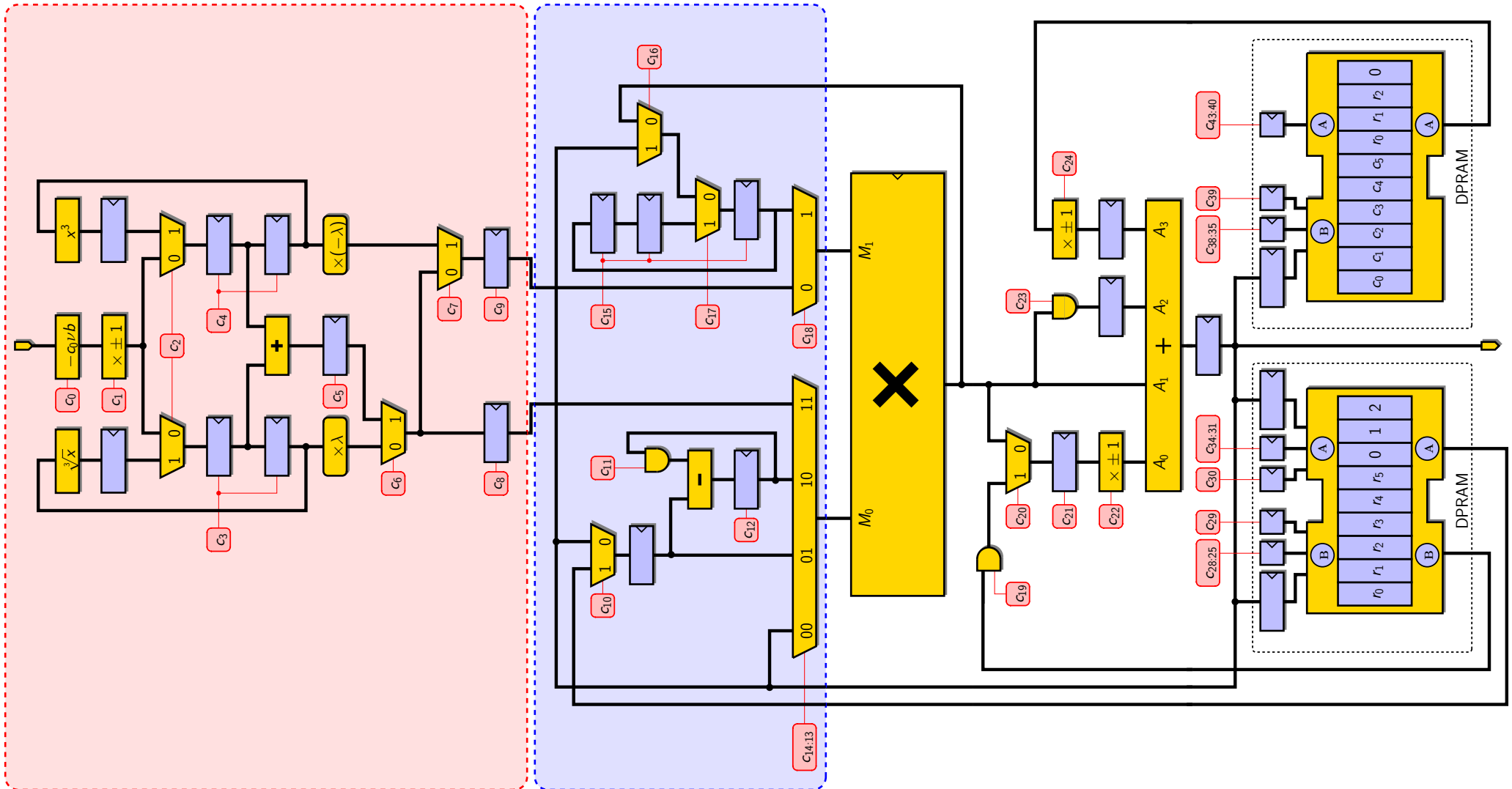
A parallel operator for the η_T pairing



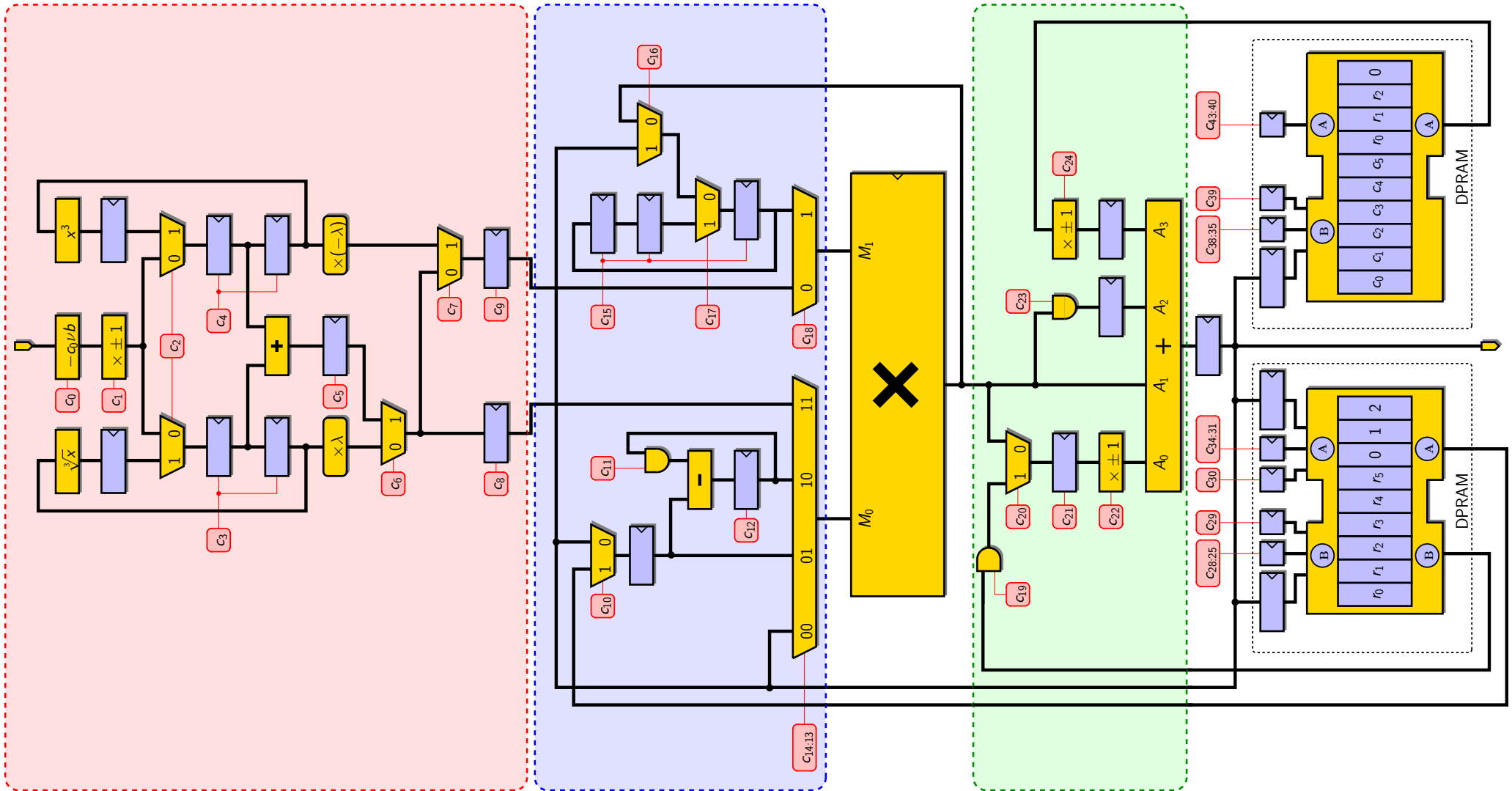
A parallel operator for the η_T pairing



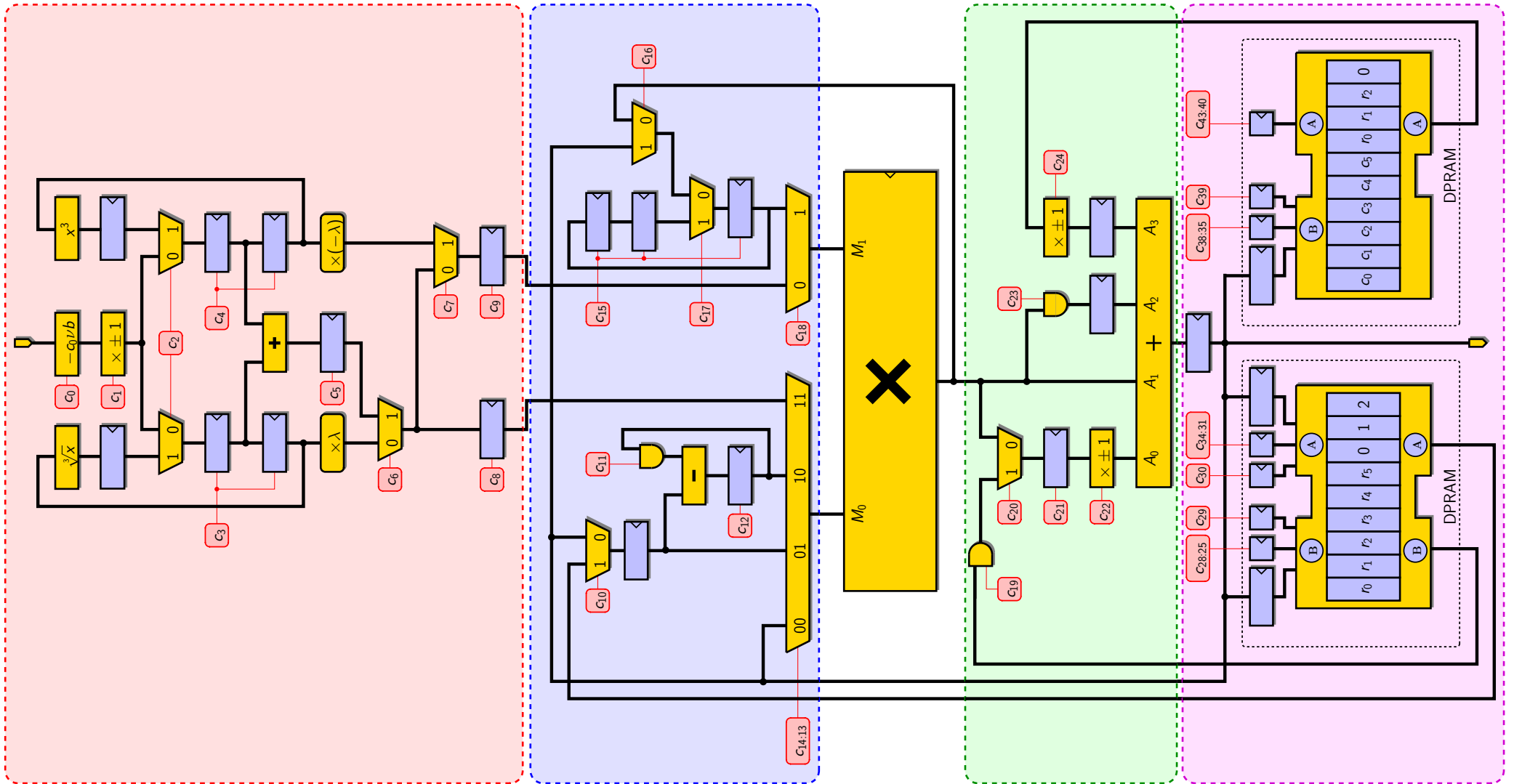
A parallel operator for the η_T pairing



A parallel operator for the η_T pairing



A parallel operator for the η_T pairing



Outline of the talk

- ▶ Previously in the Jean-Luc Beuchat Tour
- ▶ A closer look at the algorithm
- ▶ Accelerating the η_T pairing
- ▶ Accelerating the final exponentiation
- ▶ Implementation results
- ▶ Concluding thoughts

Outline of the talk

- ▶ Previously in the Jean-Luc Beuchat Tour
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- ▶ Accelerating the η_T pairing
- ▶ Accelerating the final exponentiation (in characteristic 3)
- ▶ Implementation results
- ▶ Concluding thoughts

The final exponentiation

- ▶ Compute $\hat{e}(P, Q)$ as $\eta_T(P, Q)^M$ with $\eta_T(P, Q) \in \mathbb{F}_{3^{6m}}^\times$ and

$$M = (3^{3m} - 1) (3^m + 1) \left(3^m + 1 \mp 3^{(m+1)/2} \right)$$

The final exponentiation

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- ▶ Operations over \mathbb{F}_{3^m} : $73 \times$, $3m + 3$ Frobenius, $3m + 175 +$, and 1 inversion

The final exponentiation

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- ▶ Operations over \mathbb{F}_{3^m} : $73 \times$, $3m + 3$ Frobenius, $3m + 175 +$, and 1 inversion ($\sim \log m \times$ and $m - 1$ Frobenius)

The final exponentiation

- ▶ Compute $\hat{e}(P, Q)$ as $\eta_T(P, Q)^M$ with $\eta_T(P, Q) \in \mathbb{F}_{3^{6m}}^\times$ and

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- ▶ Operations over \mathbb{F}_{3^m} : $73 \times$, $3m + 3$ Frobenius, $3m + 175 +$, and 1 inversion ($\sim \log m \times$ and $m - 1$ Frobenius)
- ▶ Cost of the η_T pairing:
 - $(m + 1)/2$ iterations
 - $17 \times$, 10 Frobenius and $38 +$ over \mathbb{F}_{3^m} per iteration

The final exponentiation

- ▶ Compute $\hat{e}(P, Q)$ as $\eta_T(P, Q)^M$ with $\eta_T(P, Q) \in \mathbb{F}_{3^m}^\times$ and

$$M = (3^{3m} - 1) (3^m + 1) \left(3^m + 1 \mp 3^{(m+1)/2} \right)$$

- ▶ Operations over \mathbb{F}_{3^m} : $73 \times$, $3m + 3$ Frobenius, $3m + 175 +$, and 1 inversion ($\sim \log m \times$ and $m - 1$ Frobenius)
- ▶ Cost of the η_T pairing:
 - $(m + 1)/2$ iterations
 - $17 \times$, 10 Frobenius and $38 +$ over \mathbb{F}_{3^m} per iteration
- ▶ The final exponentiation is **much cheaper** than the η_T pairing
- ▶ **Challenge** for the final exponentiation:
 - computation in the **same time** as the η_T pairing
 - ... using as **few resources** as possible

The final exponentiation

- ▶ First idea: use the unified operator
 - the smallest architecture supporting all the required operations over \mathbb{F}_{3^m}
 - purely sequential scheduling

The final exponentiation

- ▶ First idea: use the unified operator
 - the smallest architecture supporting all the required operations over \mathbb{F}_{3^m}
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- ▶ Example for $m = 97$:
 - computation in 1430 clock cycles

The final exponentiation

- ▶ First idea: use the unified operator
 - the smallest architecture supporting all the required operations over \mathbb{F}_{3^m}
 - purely sequential scheduling
- ▶ Example for $m = 97$:
 - computation in 1430 clock cycles
 - ... but 833 clock cycles for the η_T pairing

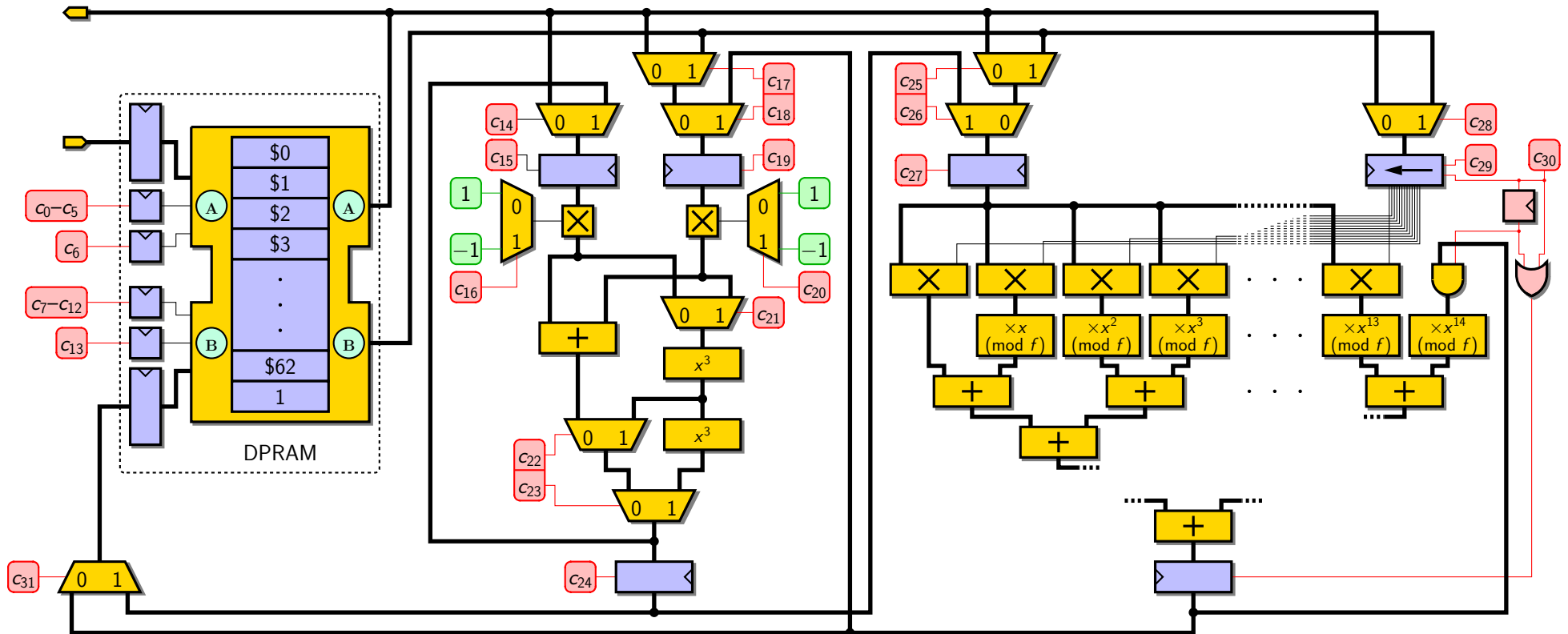
The final exponentiation

- ▶ First idea: use the unified operator
 - the smallest architecture supporting all the required operations over \mathbb{F}_{3^m}
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- ▶ Example for $m = 97$:
 - computation in 1430 clock cycles
 - ... but 833 clock cycles for the η_T pairing
- ▶ Some parallelism is required

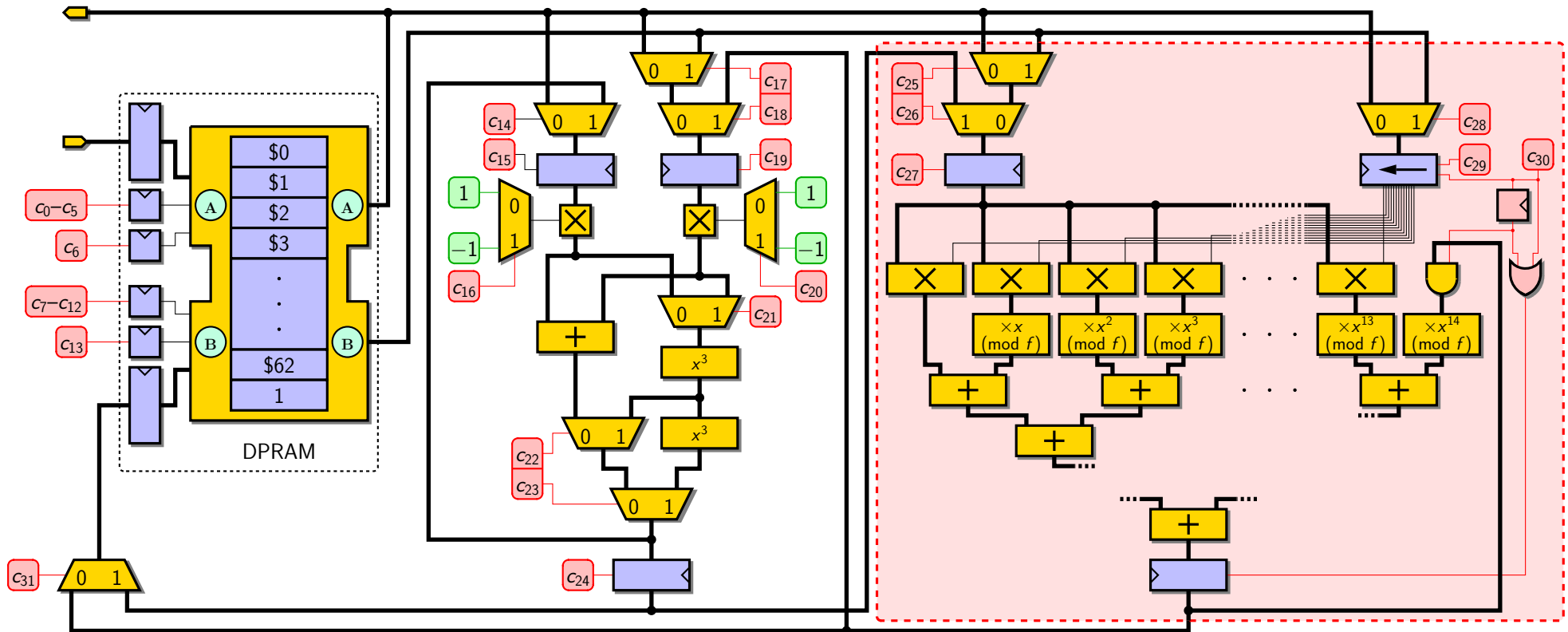
The final exponentiation

- ▶ First idea: use the unified operator
 - the smallest architecture supporting all the required operations over \mathbb{F}_{3^m}
 - purely sequential scheduling
- ▶ Example for $m = 97$:
 - computation in 1430 clock cycles
 - ... but 833 clock cycles for the η_T pairing
- ▶ Some parallelism is required
- ▶ New coprocessor with two arithmetic units:
 - a standalone multiplier, based on a parallel-serial scheme
 - a unified operator supporting addition/subtraction, Frobenius map and double Frobenius map

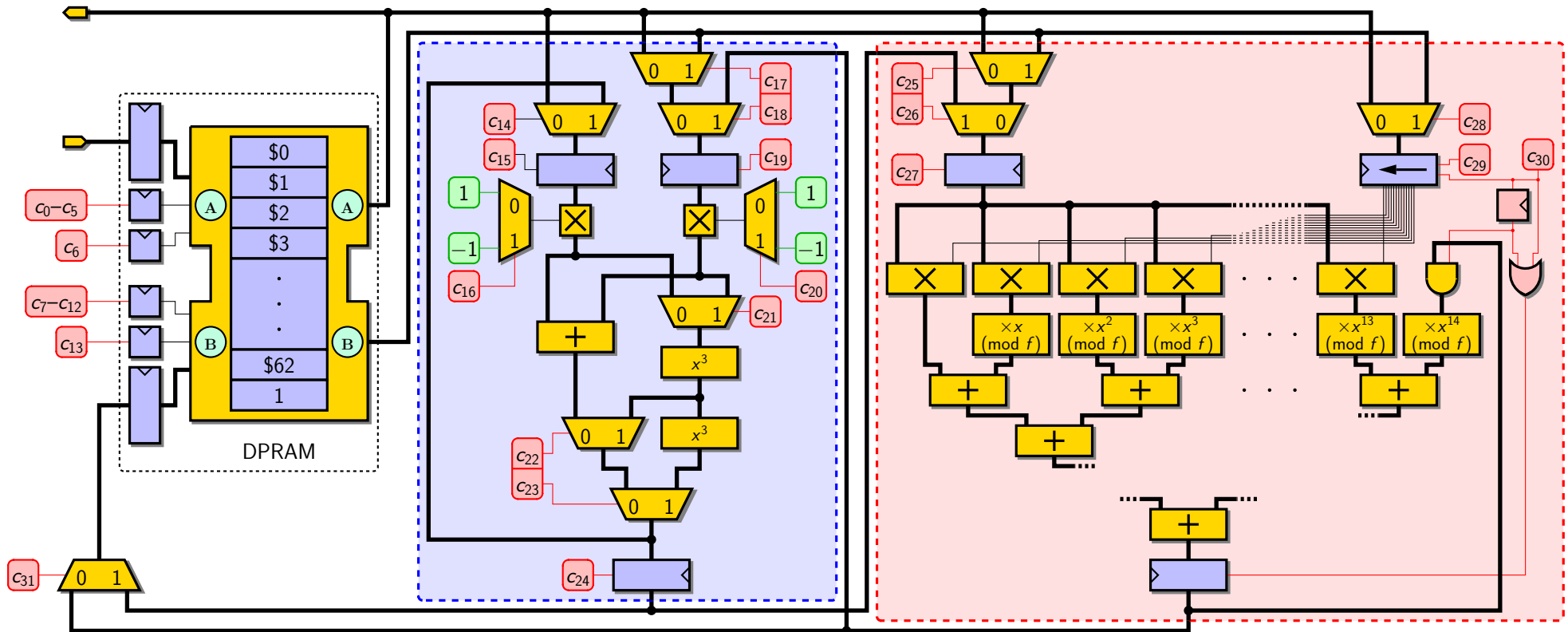
A coprocessor for the final exponentiation



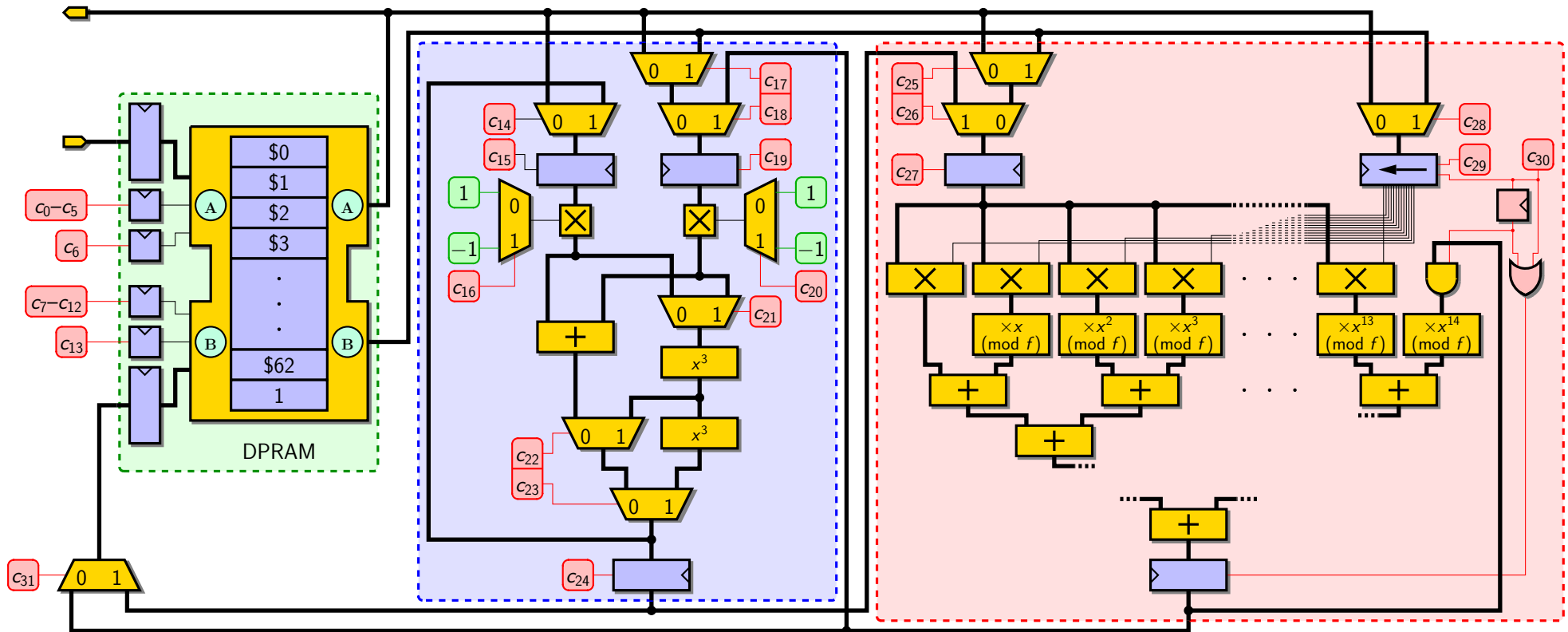
A coprocessor for the final exponentiation



A coprocessor for the final exponentiation



A coprocessor for the final exponentiation



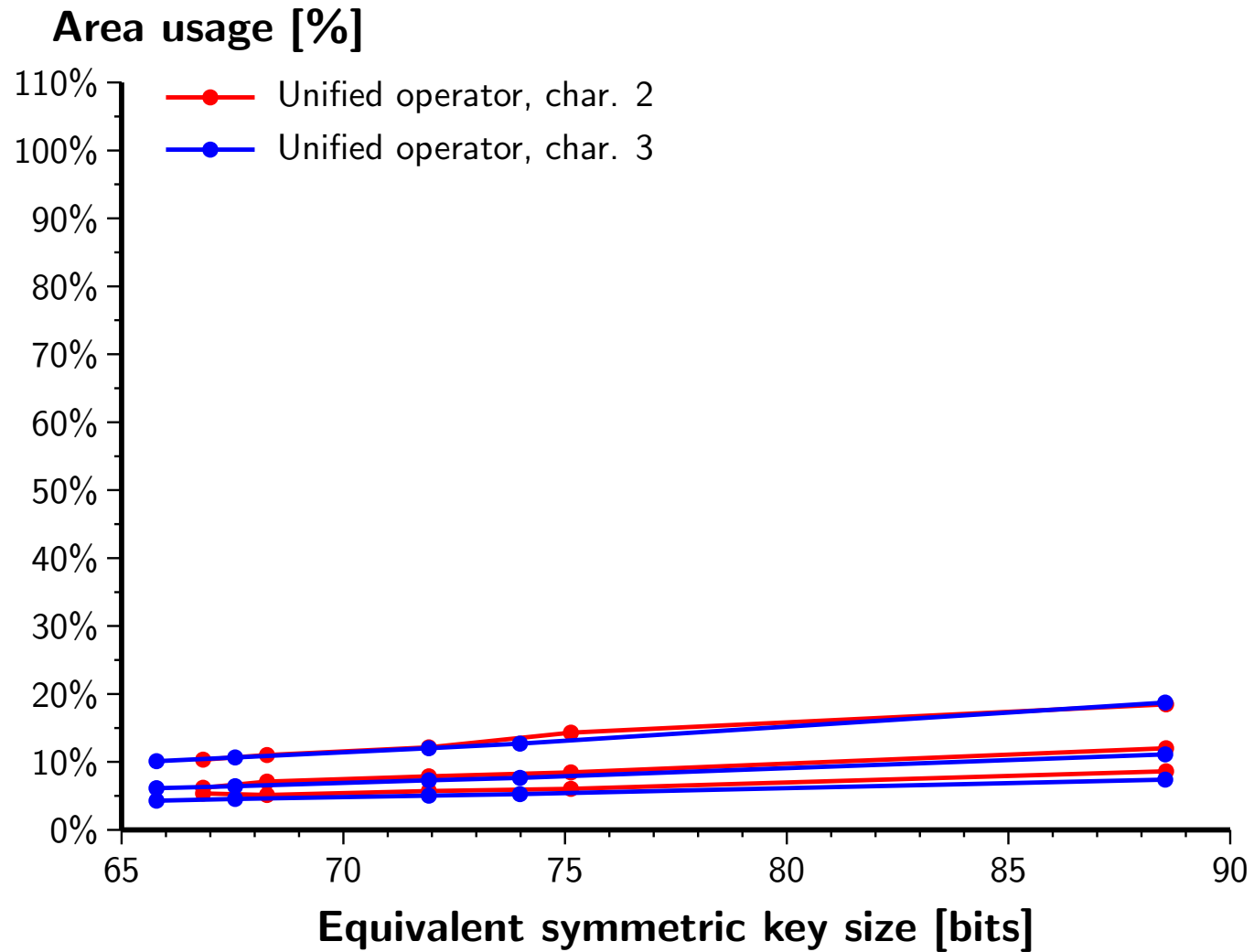
Outline of the talk

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- ▶ **Implementation results**
- ▶ Concluding thoughts

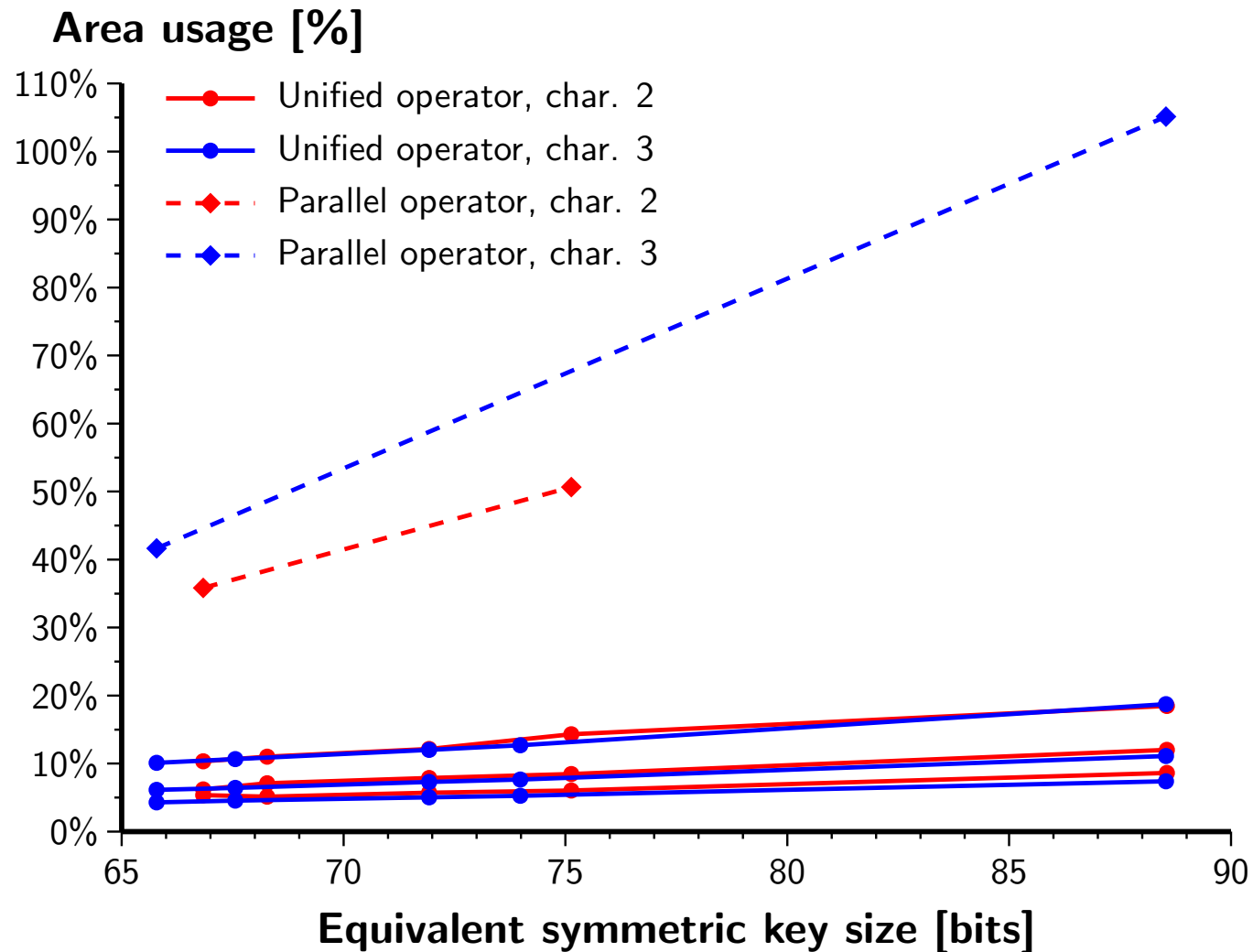
Experimental setup

- ▶ Full coprocessor for computation of the Tate pairing
- ▶ Architecture based on the two parallel accelerators
- ▶ Prototyped on a Xilinx Virtex-II Pro 50 FPGA (larger model)
- ▶ Post place-and-route results: area, computation time, AT product

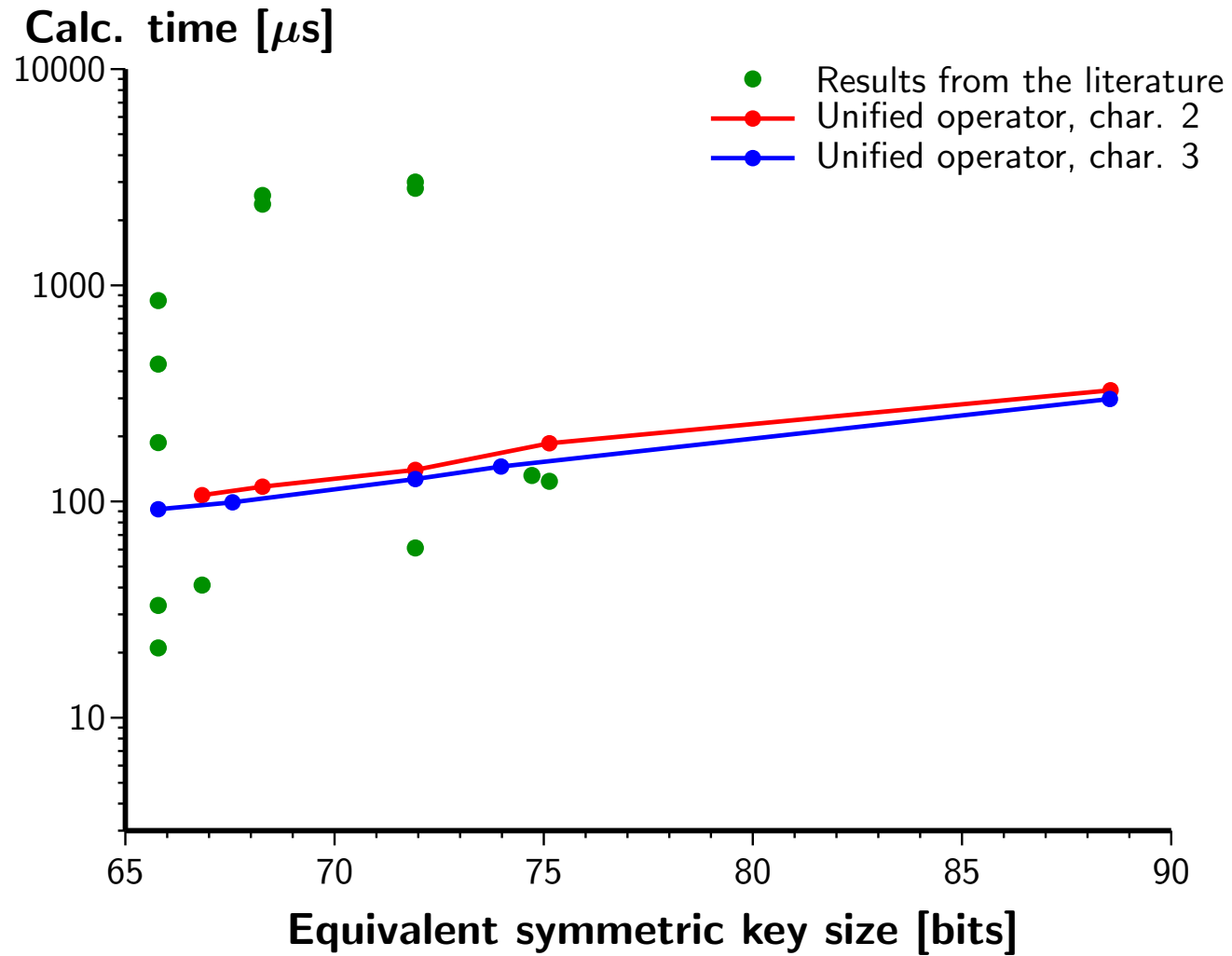
Coprocessor area



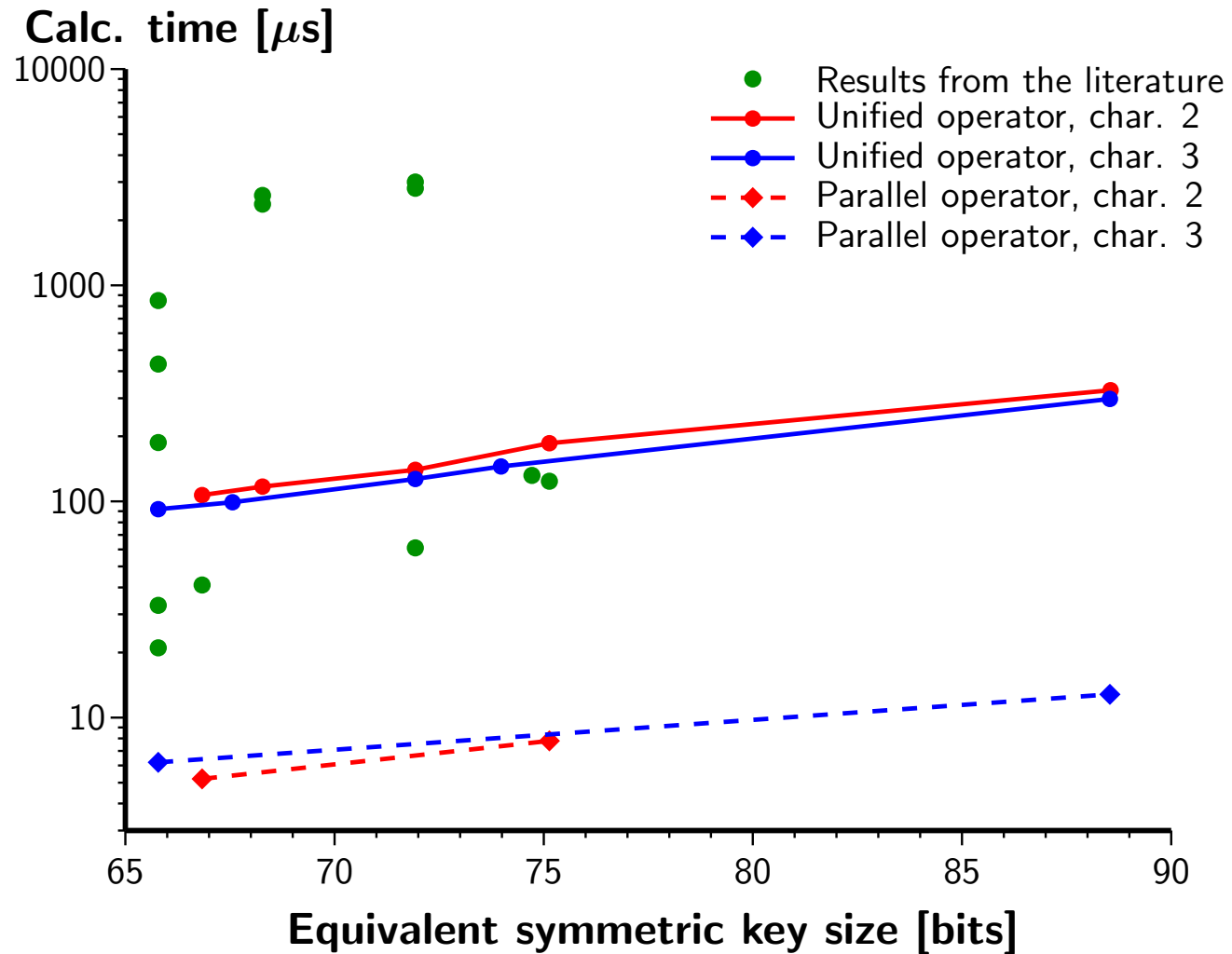
Coprocessor area



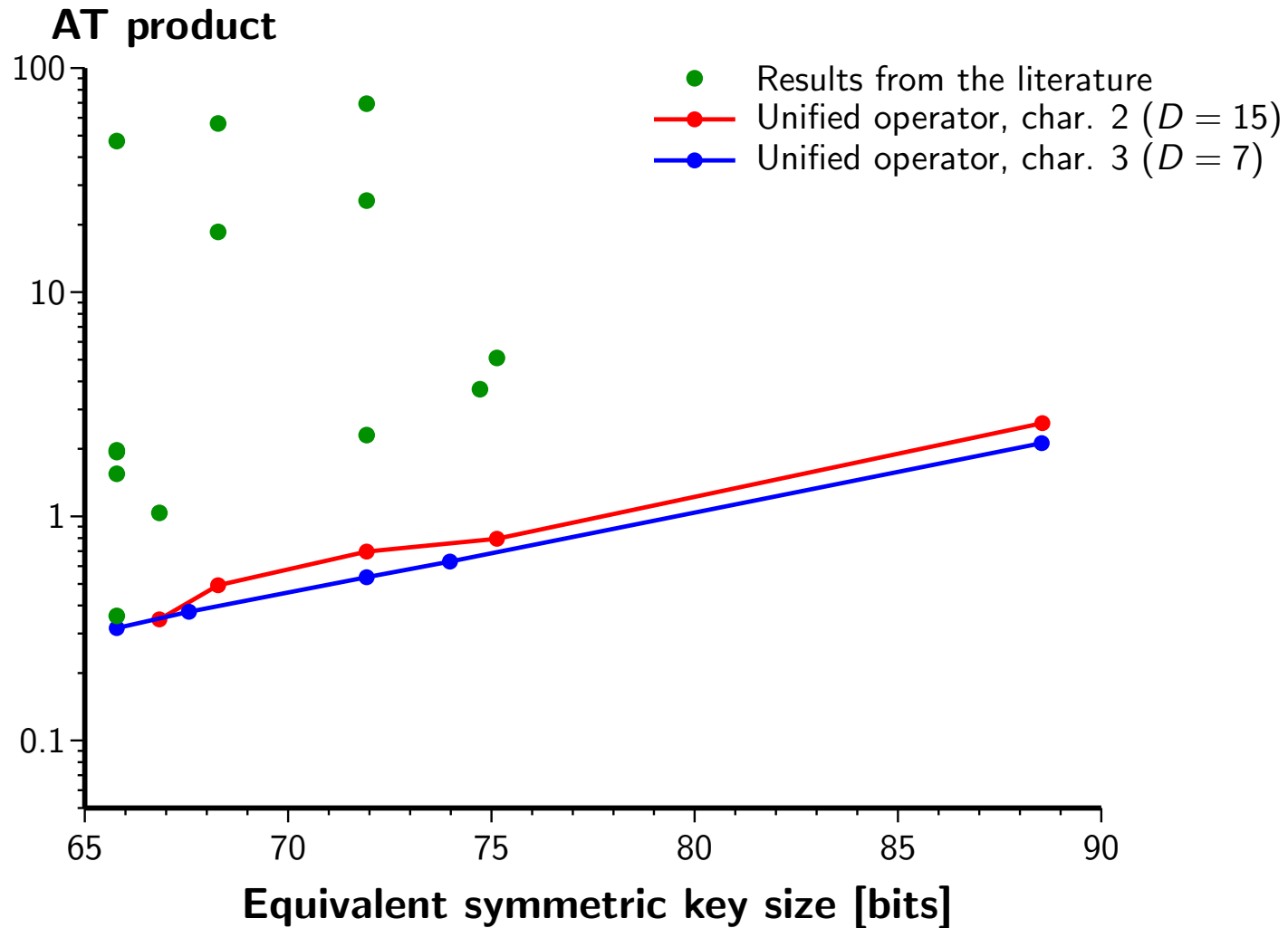
Calculation time



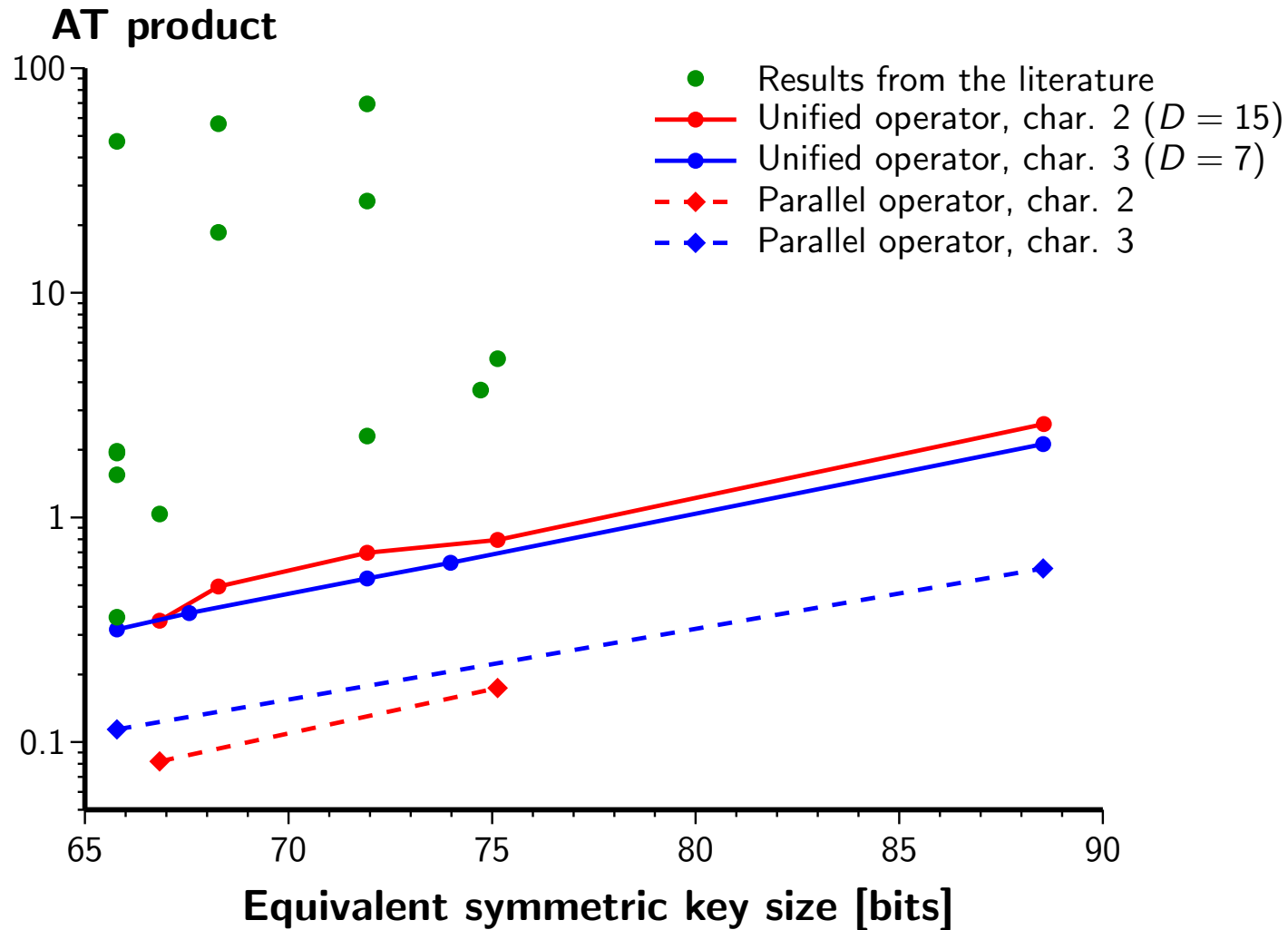
Calculation time



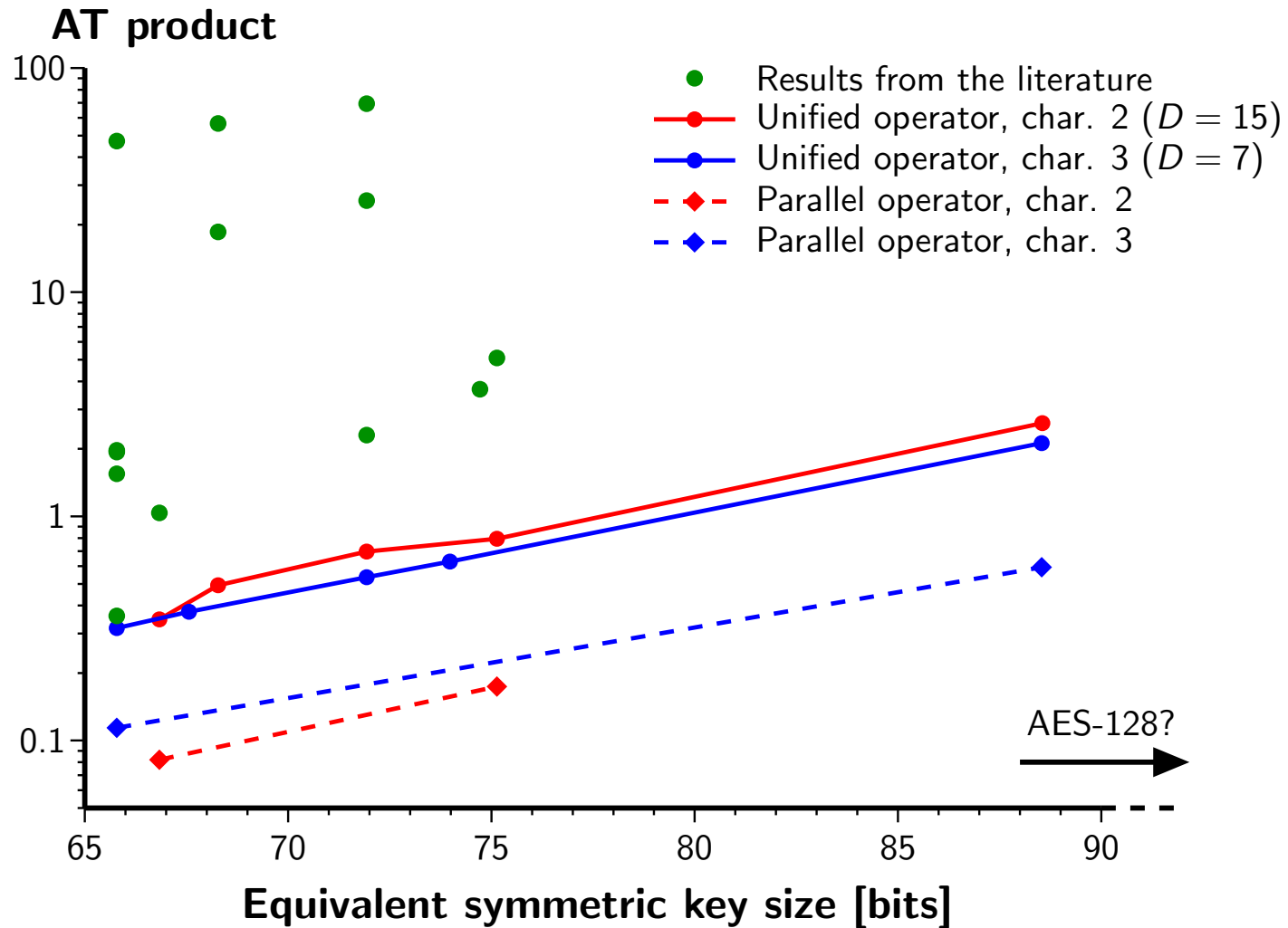
Area-time product



Area-time product



Area-time product



Outline of the talk

- ▶ Previously in the Jean-Luc Beuchat Tour
- ▶ A closer look at the algorithm
- ▶ Accelerating the η_T pairing
- ▶ Accelerating the final exponentiation
- ▶ Implementation results
- ▶ **Concluding thoughts**

Conclusion and perspectives

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Conclusion and perspectives

- ▶ The fastest implementation of the literature!
- ▶ Importance of the adequation between algorithm and architecture
- ▶ Scalability? **AES-128?**

With thanks to our sponsor



Thank you for your attention

Questions?