

Hardware Operators for Pairing-Based Cryptography

— Part I: Because size matters —

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Arénaire, LIP, ÉNS Lyon, France

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LCIS, University of Tsukuba, Japan

CSD, IPN, Mexico City, Mexico

Outline of the talk

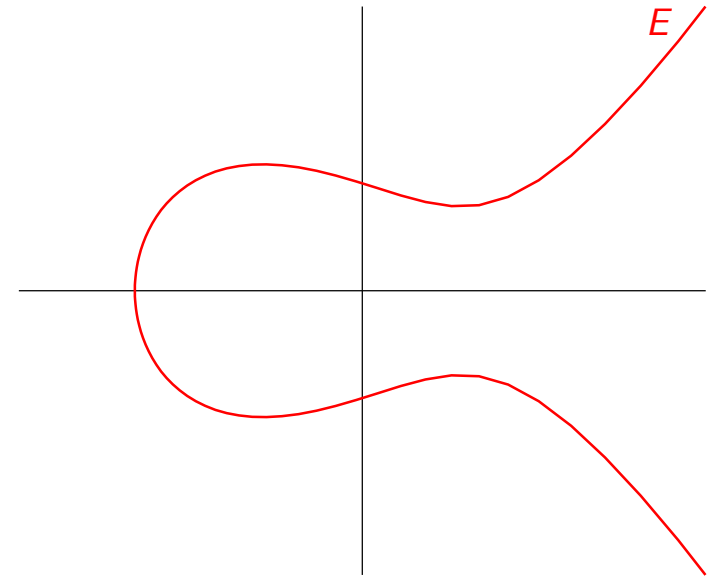
- ▶ Pairing-based cryptography
- ▶ Pairings over elliptic curves
- ▶ Finite-field arithmetic
- ▶ Implementation results
- ▶ Concluding thoughts

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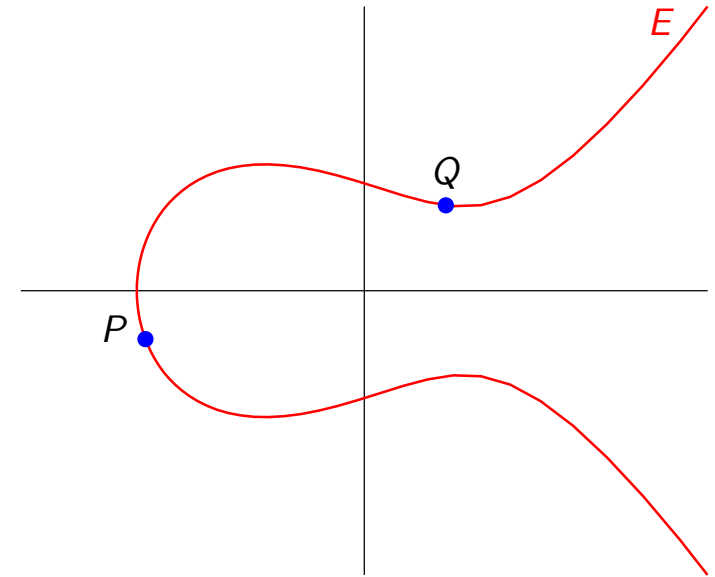
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- ▶ E defined by a Weierstraß equation of the form
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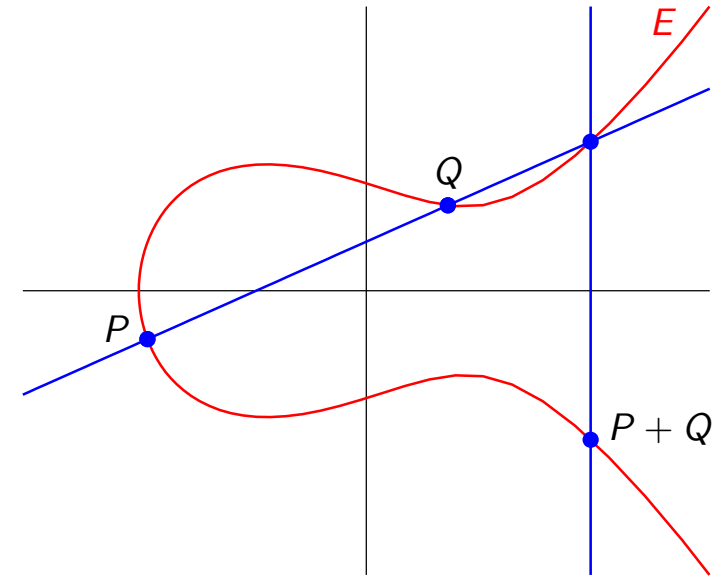
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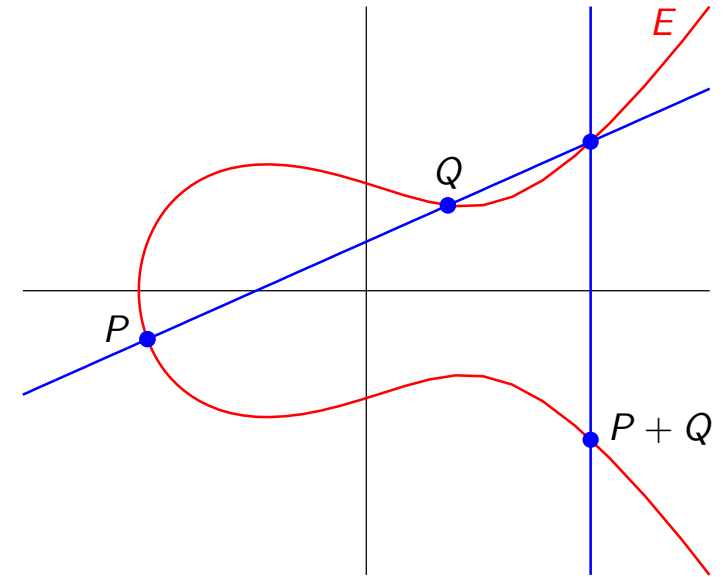
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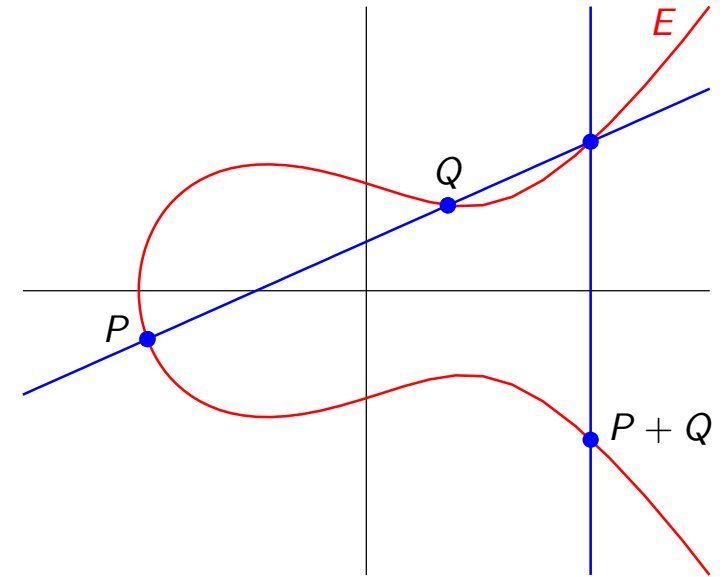
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- ▶ But there's more: bilinear pairings



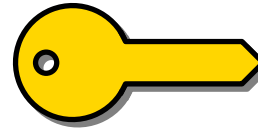
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- ▶ $(\mathbb{G}_1, +)$, an additively-written cyclic group of prime order $\#\mathbb{G}_1 = \ell$

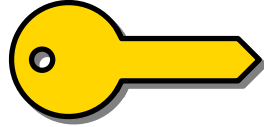
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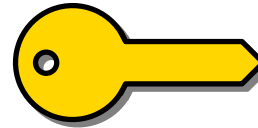
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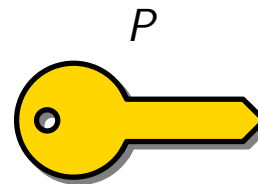
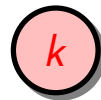
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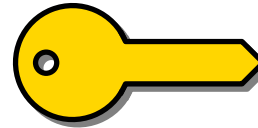
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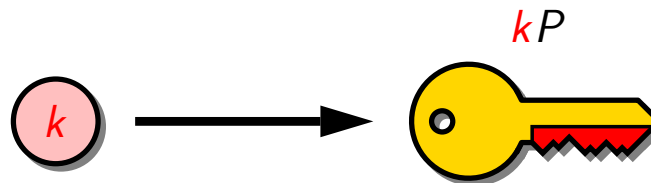
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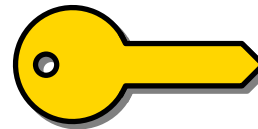
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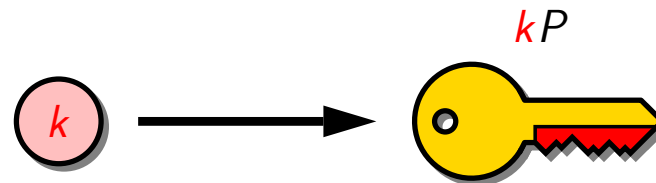
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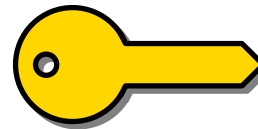


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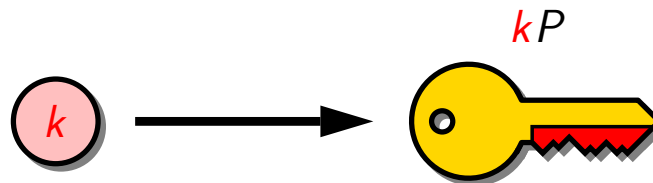
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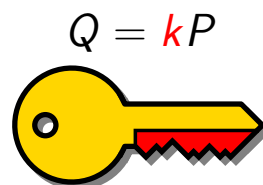
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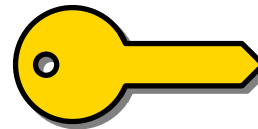
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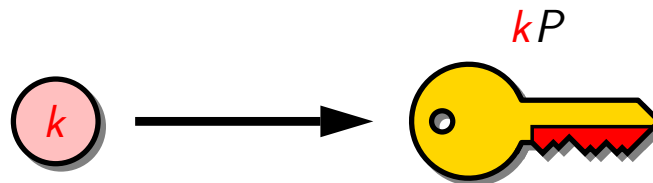
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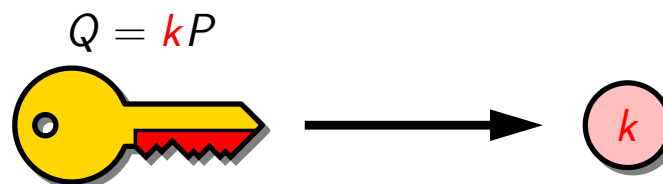
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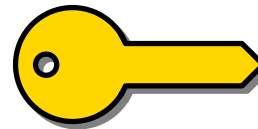
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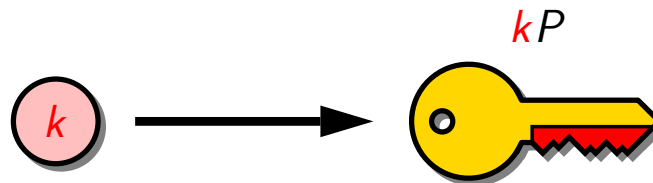
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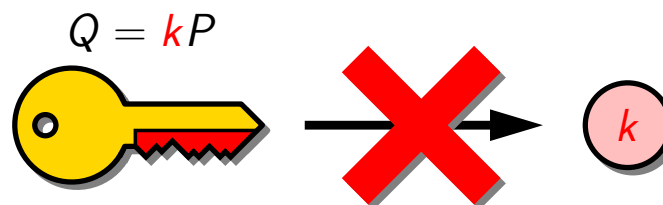
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- ▶ We assume that the discrete logarithm problem (DLP) in \mathbb{G}_1 is hard

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that satisfies the following conditions:

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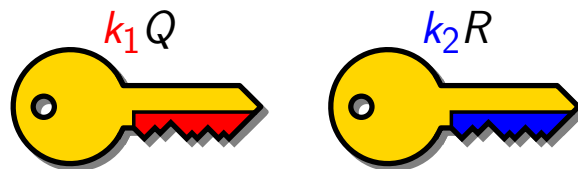
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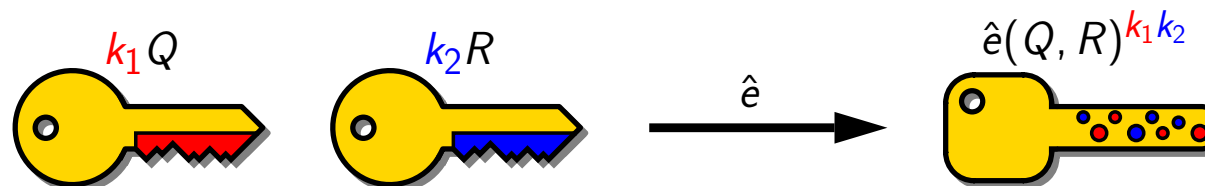
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- ▶ One-round three-party key agreement (Joux, 2000)

- ▶ Identity-based encryption

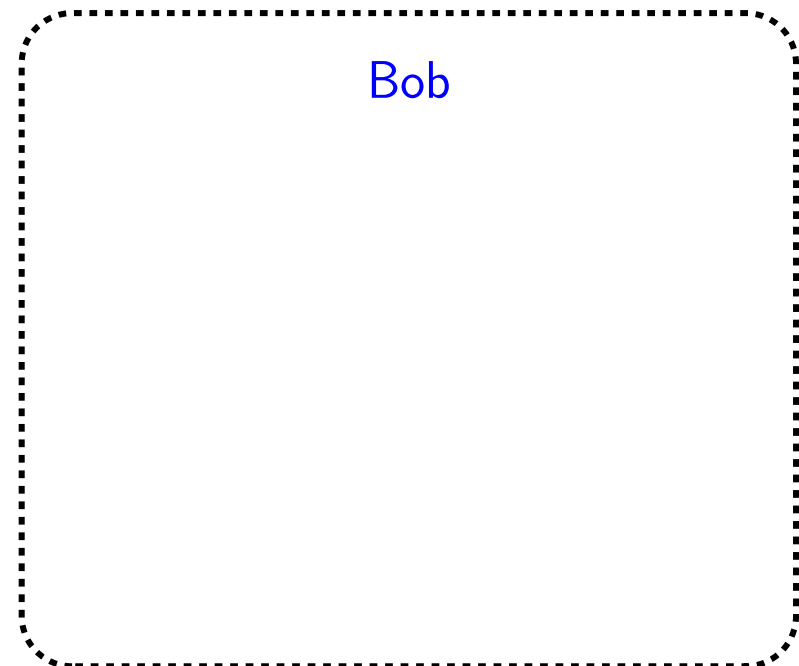
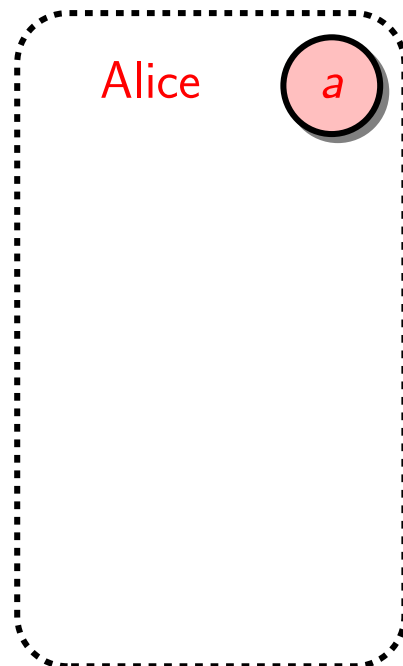
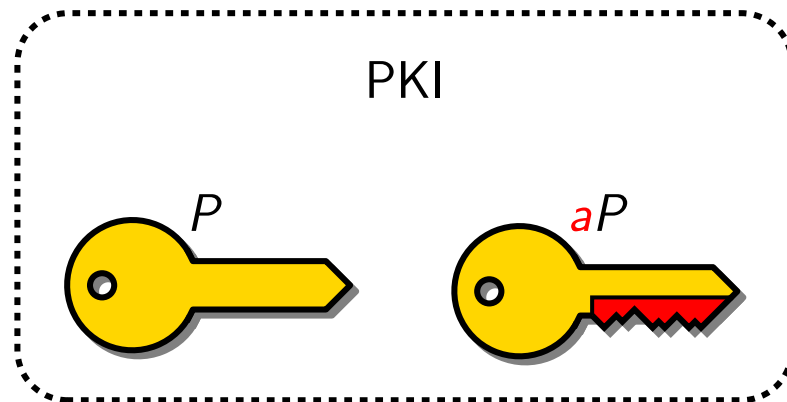
- Boneh-Franklin, 2001
- Sakai-Kasahara, 2001

- ▶ Short digital signatures

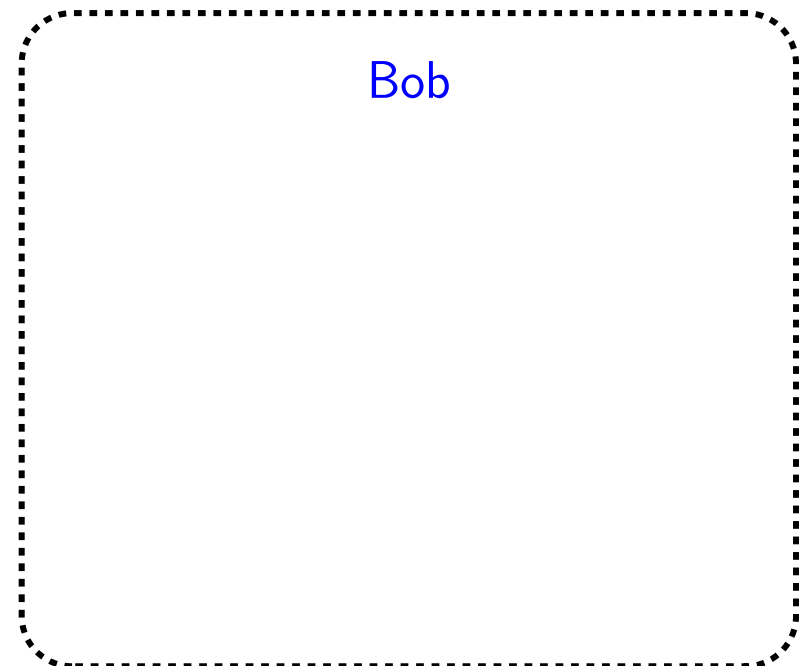
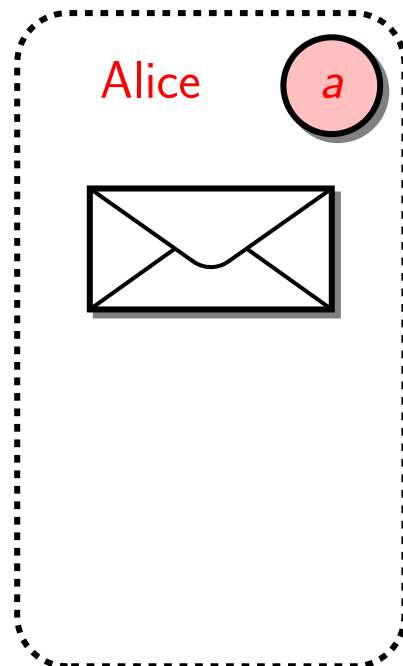
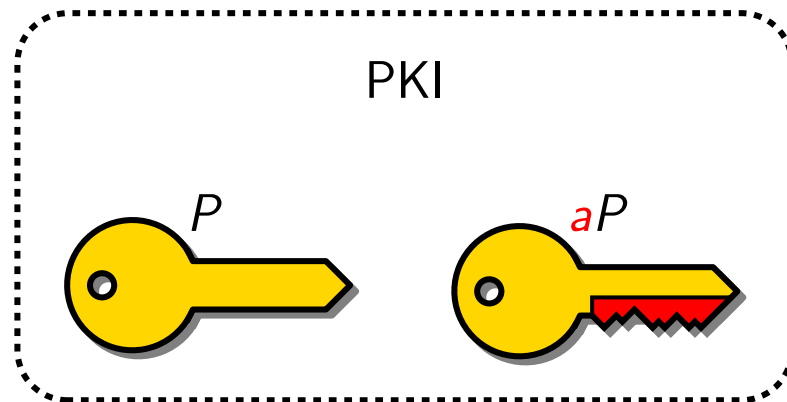
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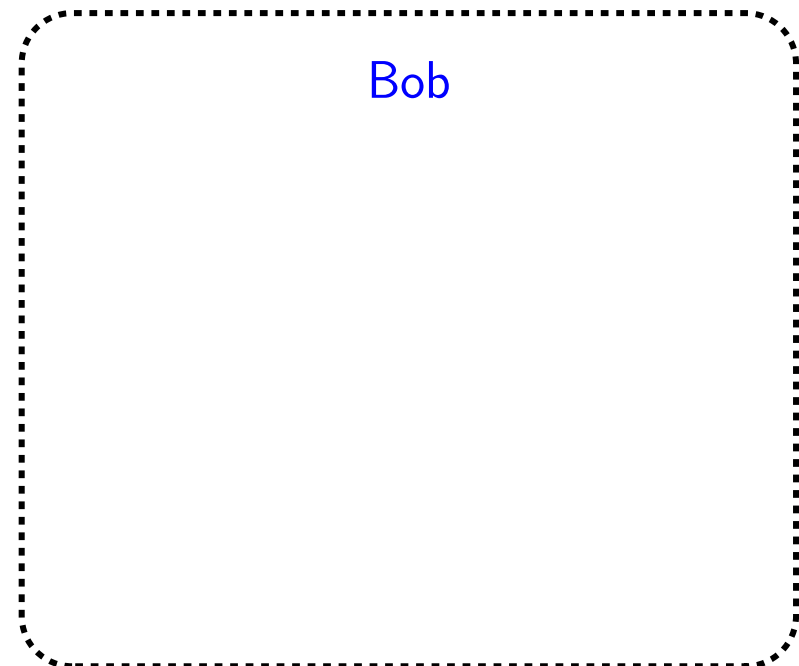
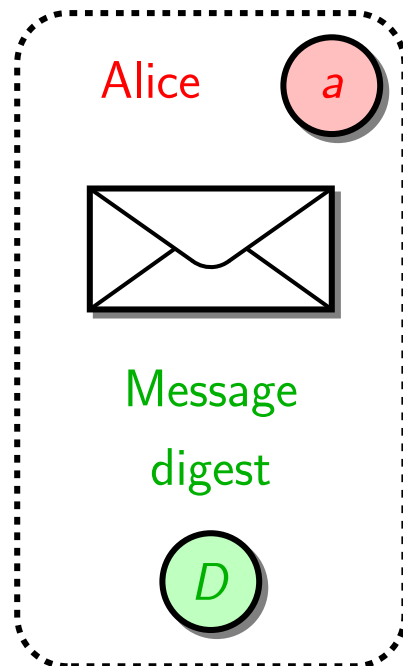
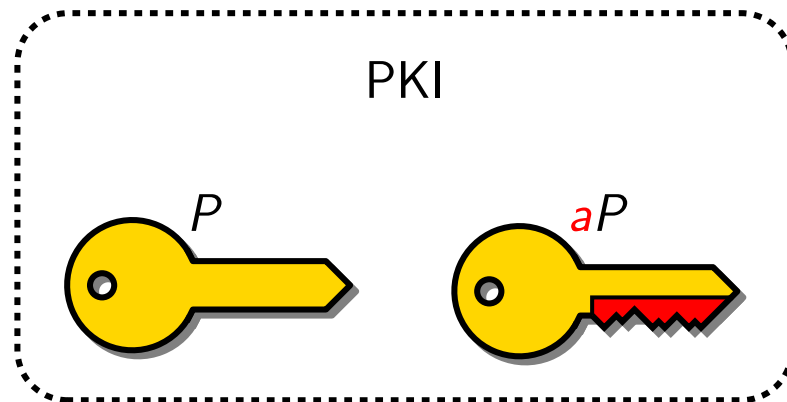
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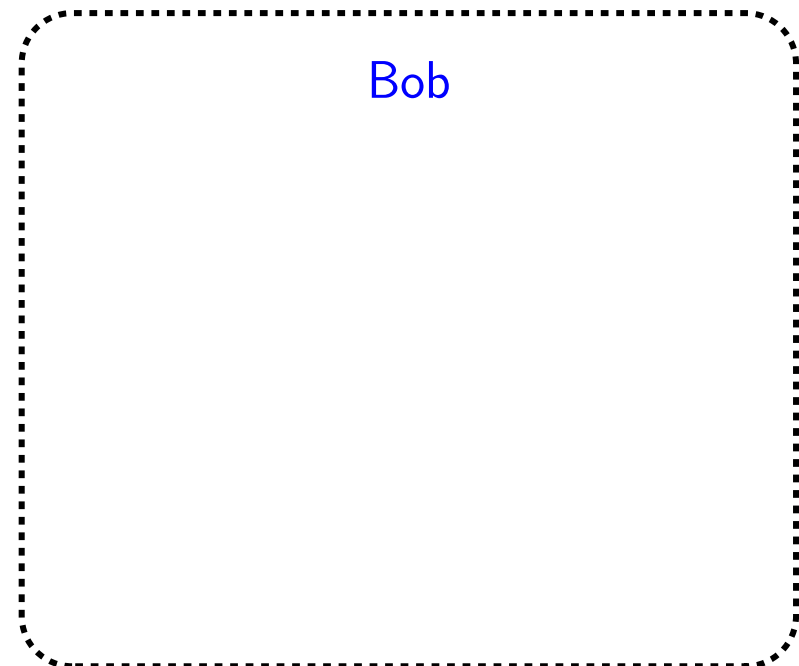
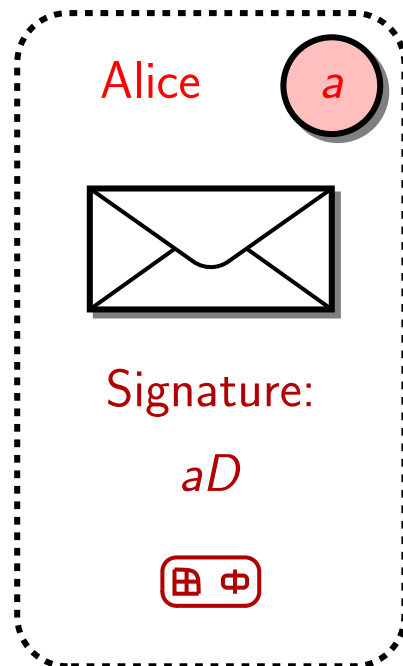
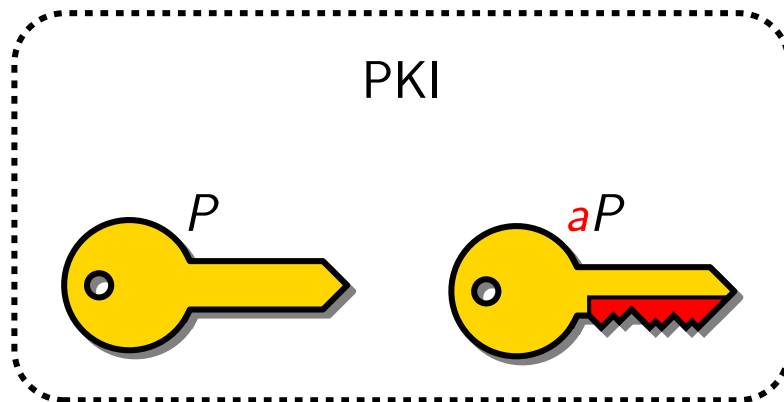
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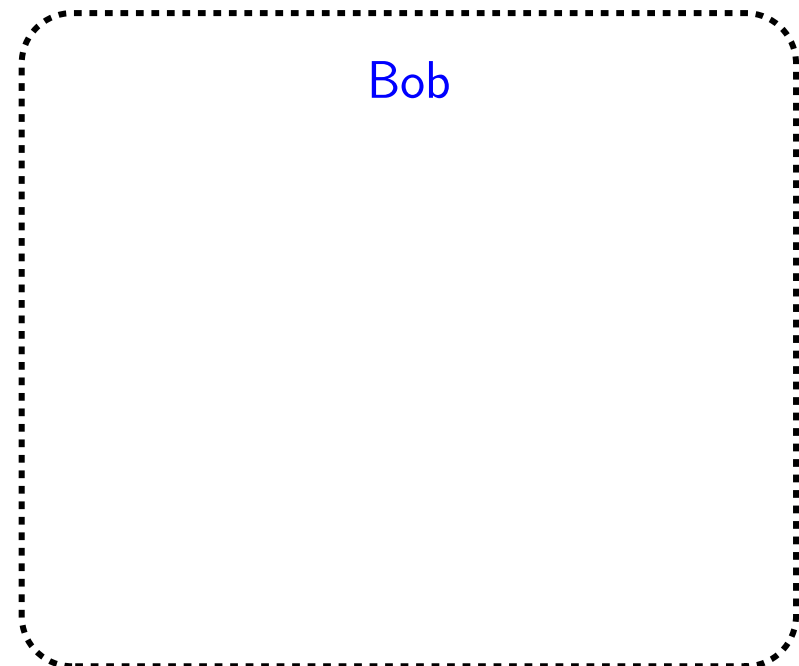
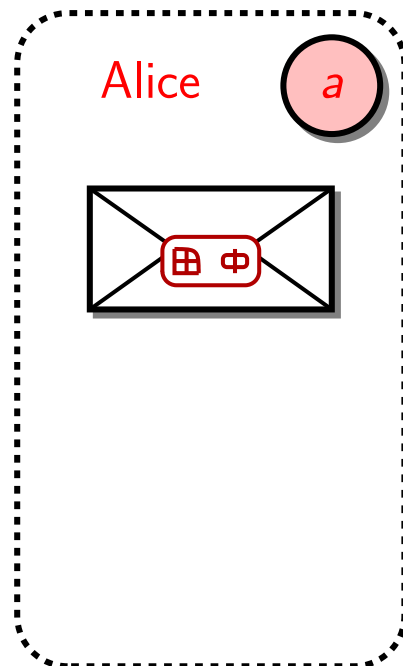
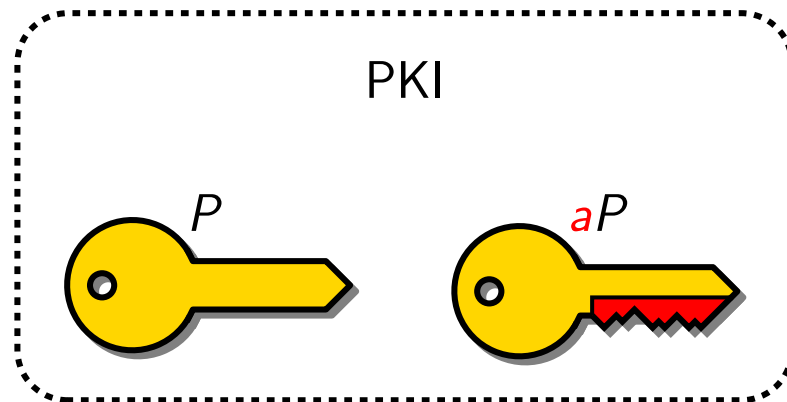
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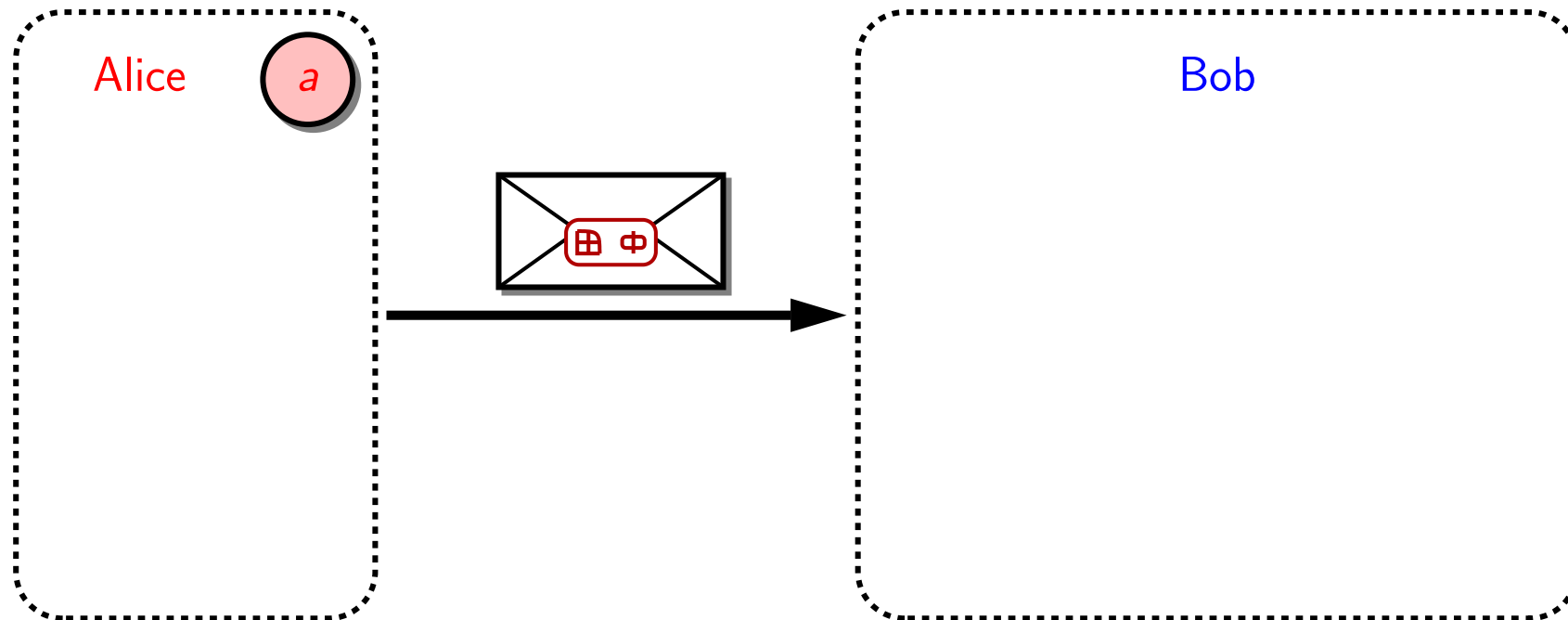
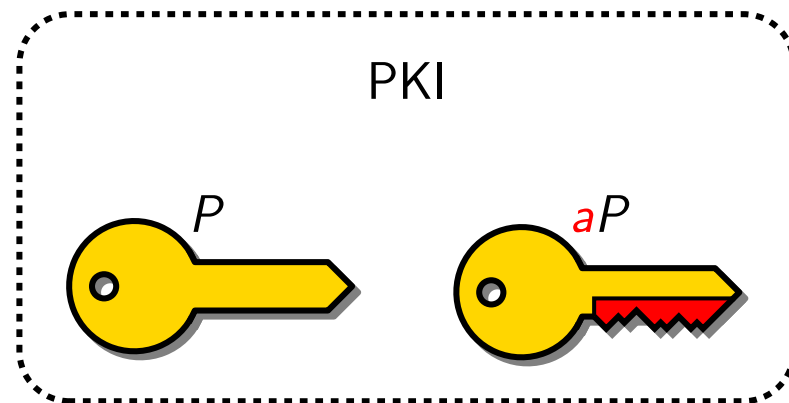
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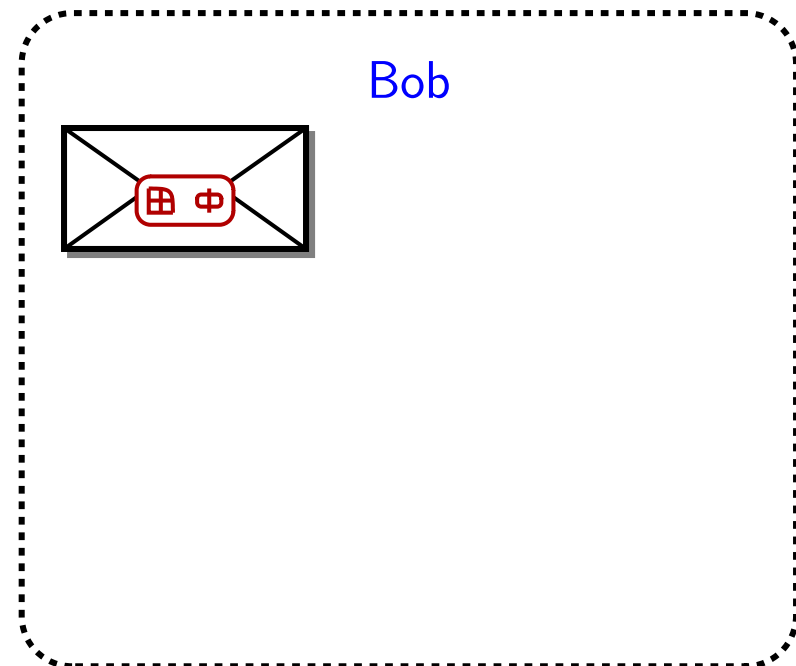
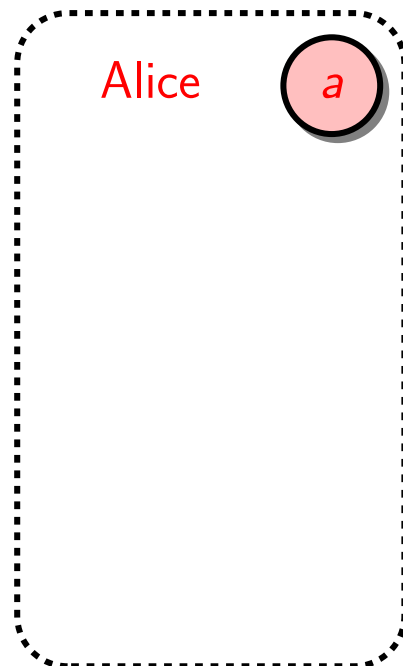
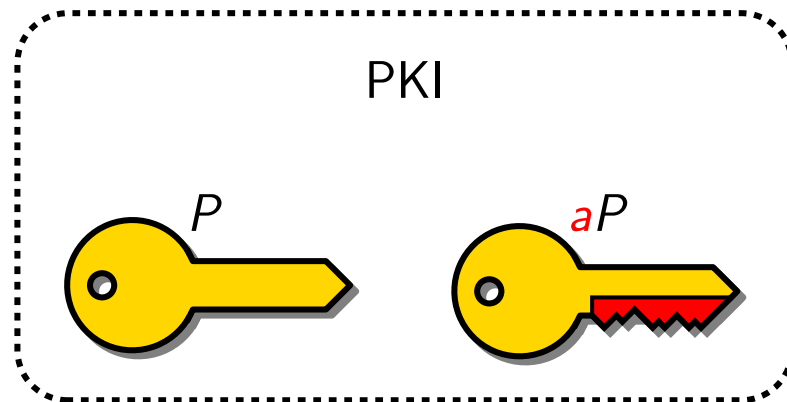
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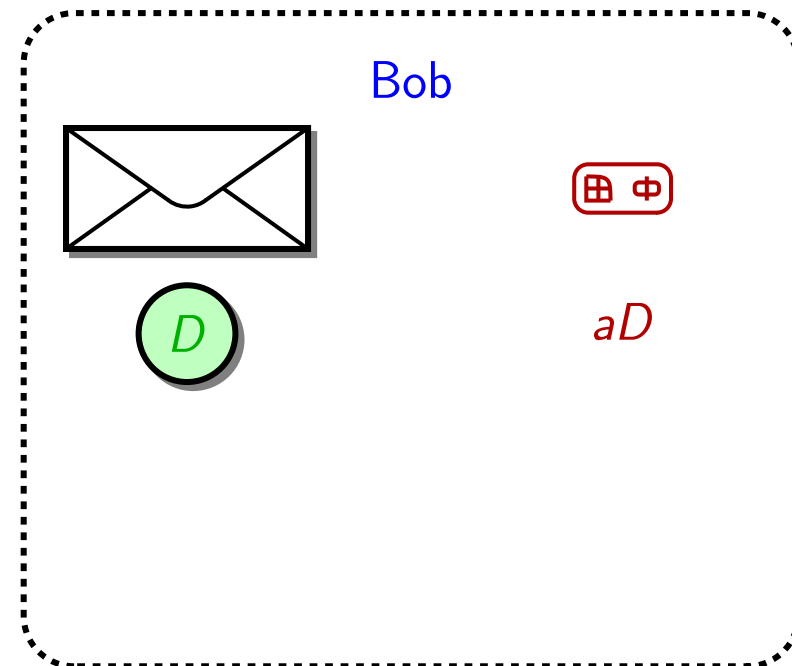
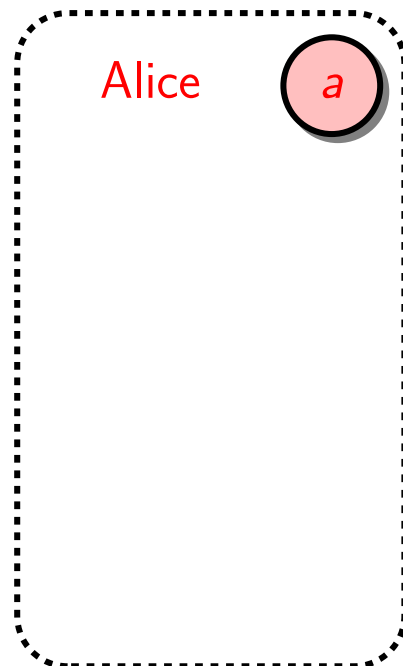
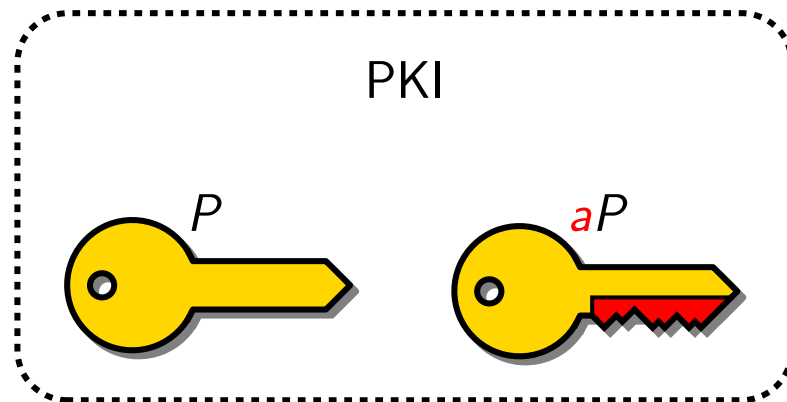
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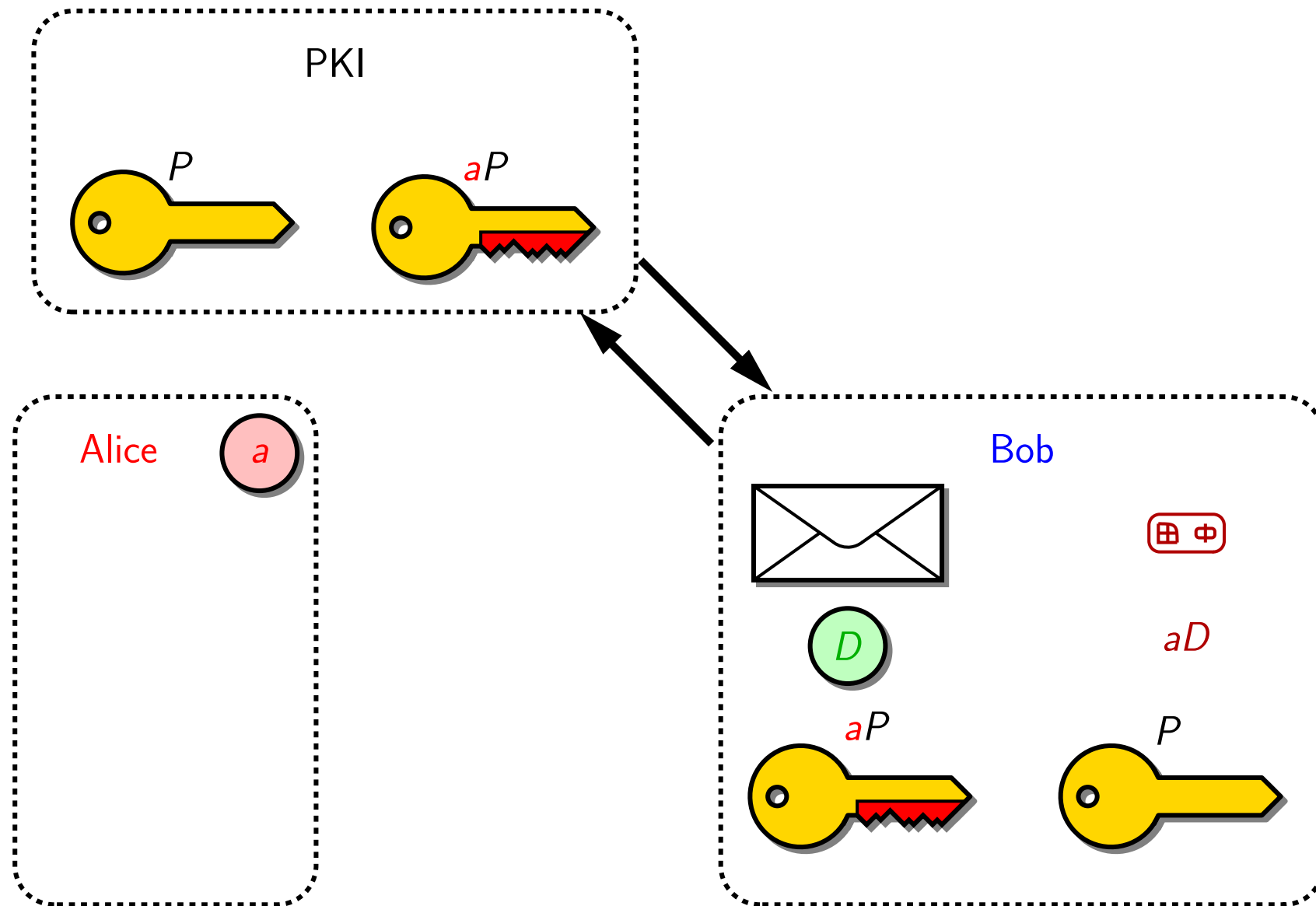
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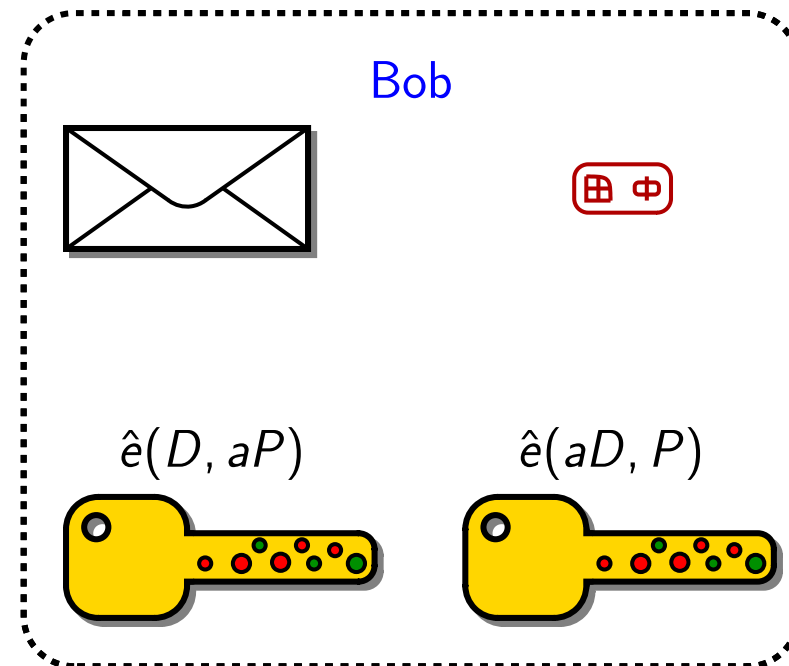
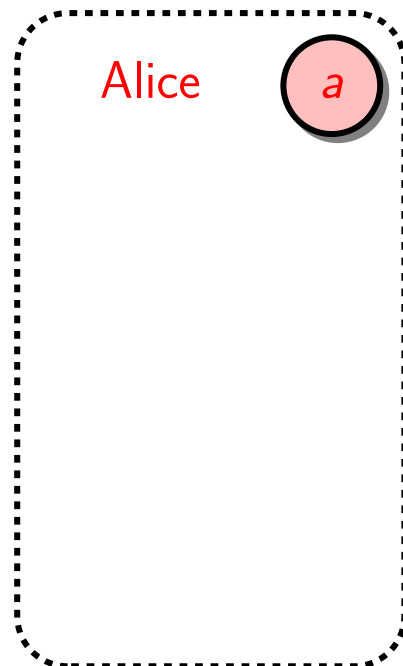
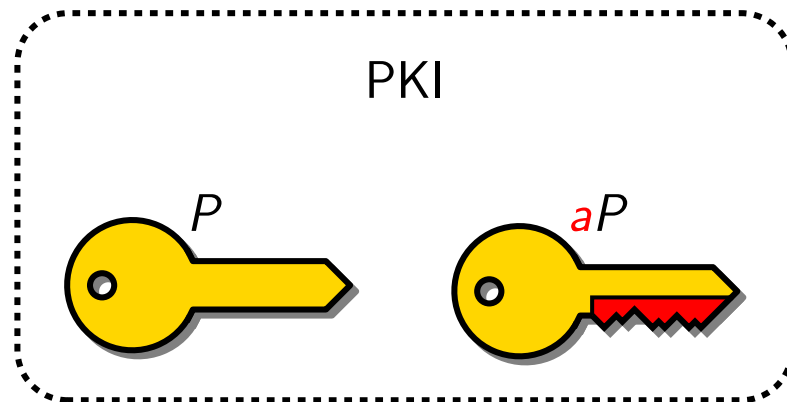
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Pairings over elliptic curves

- ▶ We first define
 - \mathbb{F}_q , a finite field, with $q = 2^m, 3^m$ or p
 - E , an elliptic curve defined over \mathbb{F}_q
 - ℓ , a large prime factor of $\#E(\mathbb{F}_q)$

Pairings over elliptic curves

- ▶ We first define
 - \mathbb{F}_q , a finite field, with $q = 2^m, 3^m$ or p
 - E , an elliptic curve defined over \mathbb{F}_q
 - ℓ , a large prime factor of $\#E(\mathbb{F}_q)$
- ▶ $\mathbb{G}_1 = E(\mathbb{F}_q)[\ell]$, the \mathbb{F}_q -rational ℓ -torsion of E :

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- bounded in the case of **supersingular elliptic curves**
(4 in characteristic 2; 6 in characteristic 3; and 2 in characteristic > 3)

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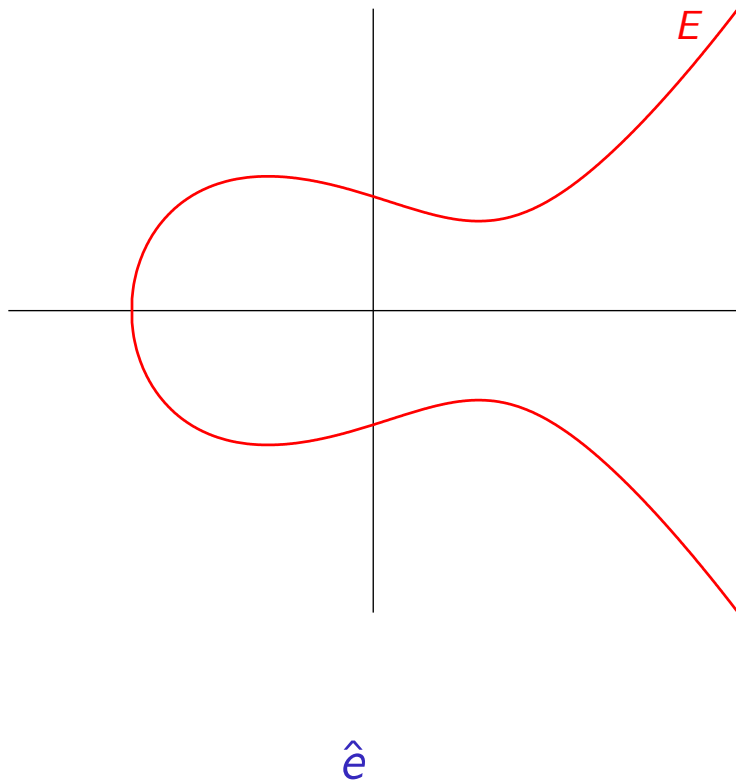
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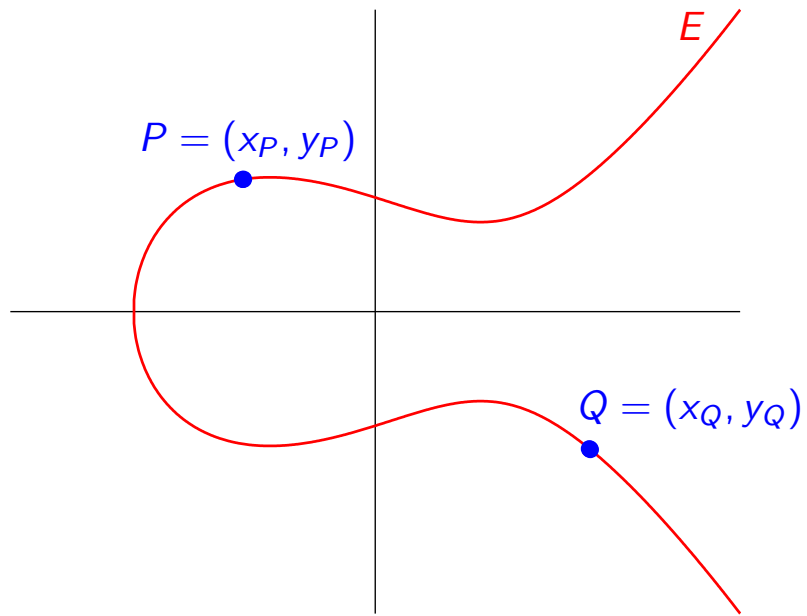
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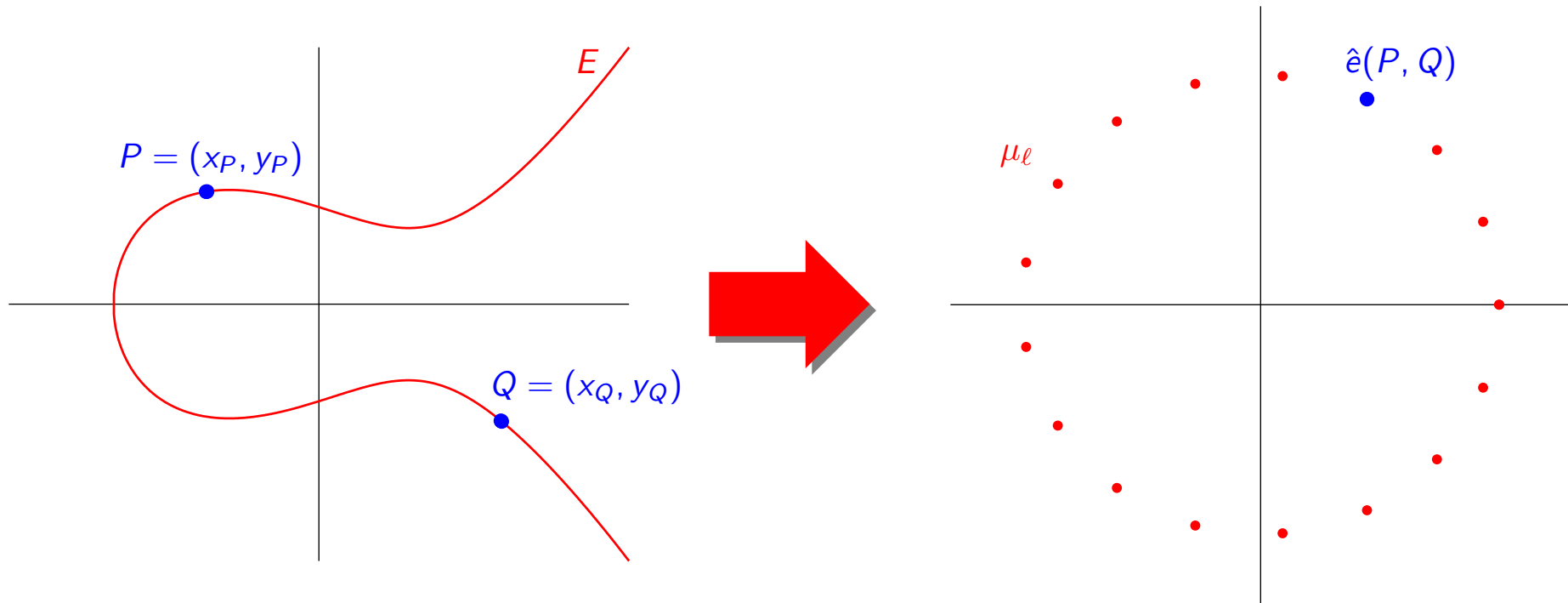


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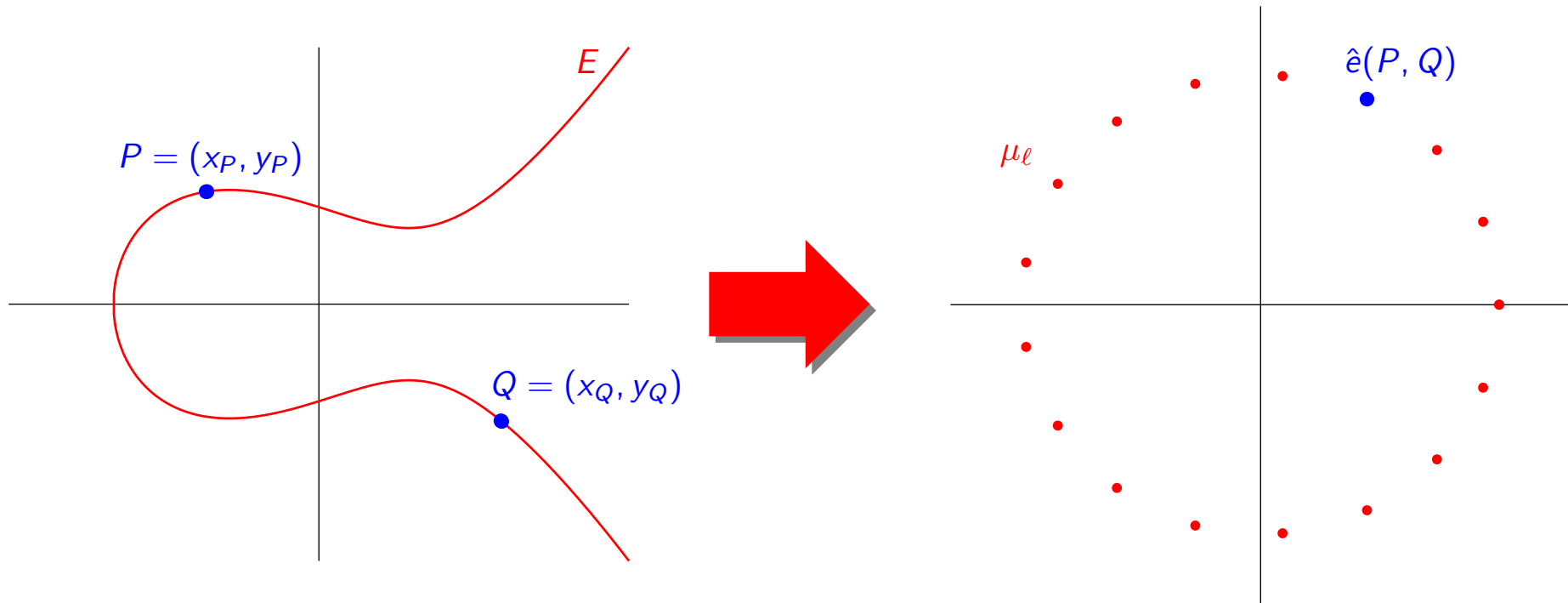
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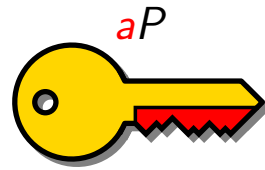
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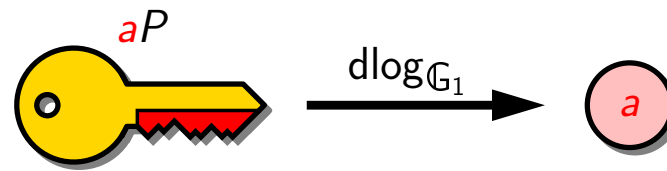
► Computation via Miller's iterative algorithm:

- $m/2$ iterations over \mathbb{F}_{2^m} and \mathbb{F}_{3^m} (η_T pairing)
- $\log_2 p$ iterations over \mathbb{F}_p

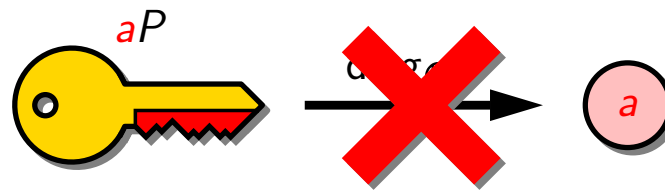
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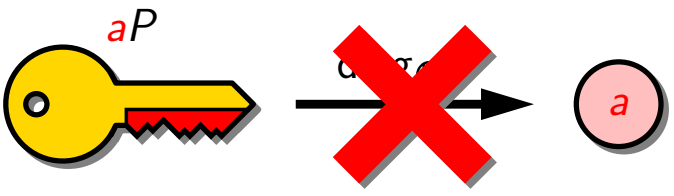


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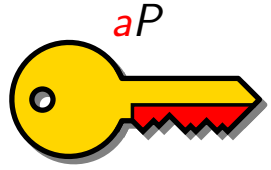


- ▶ Discrete logarithm problem should be hard in \mathbb{G}_1

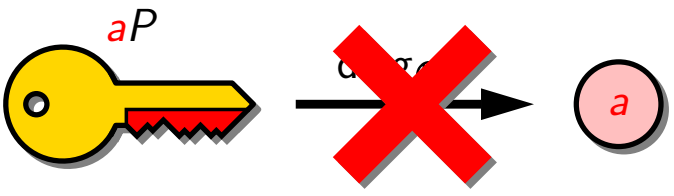
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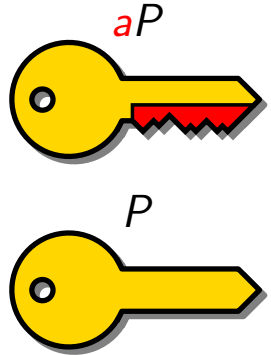
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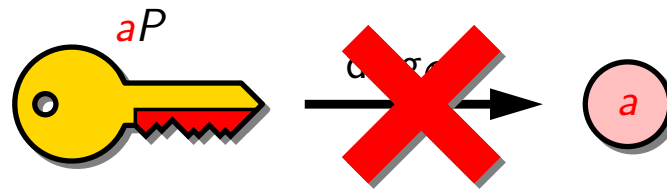
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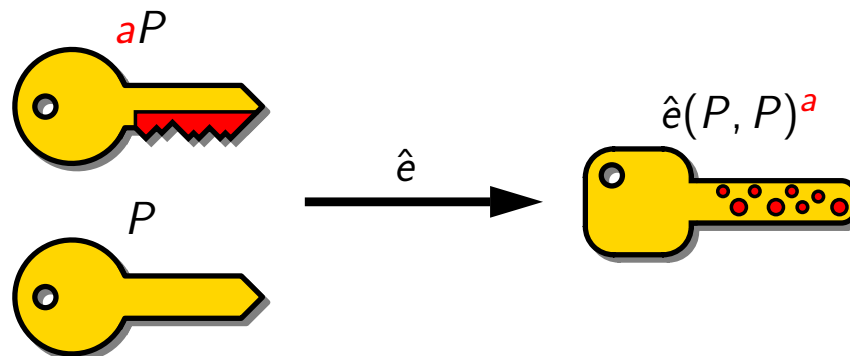
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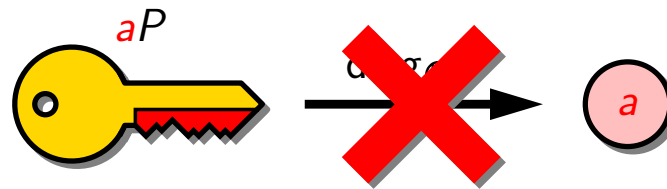
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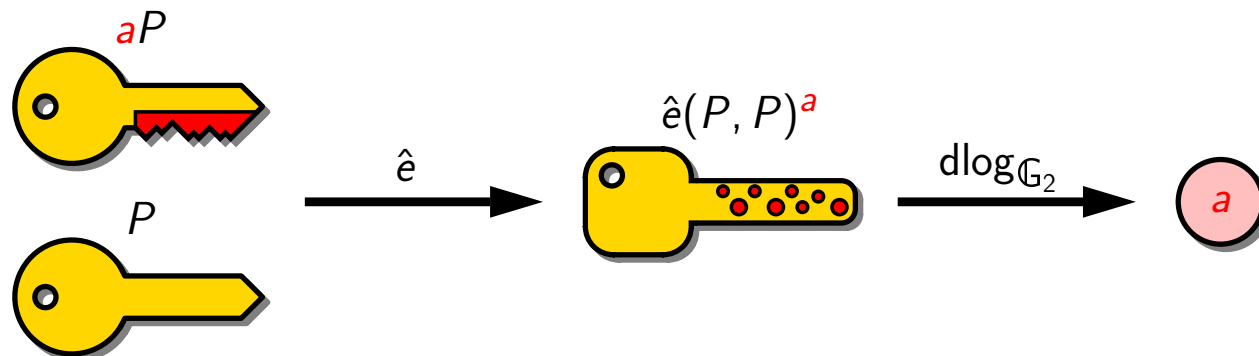
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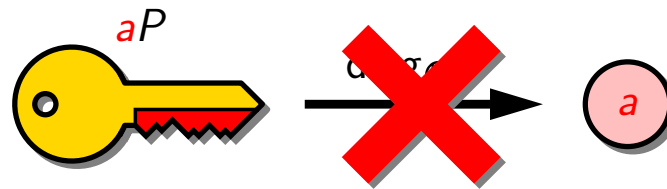
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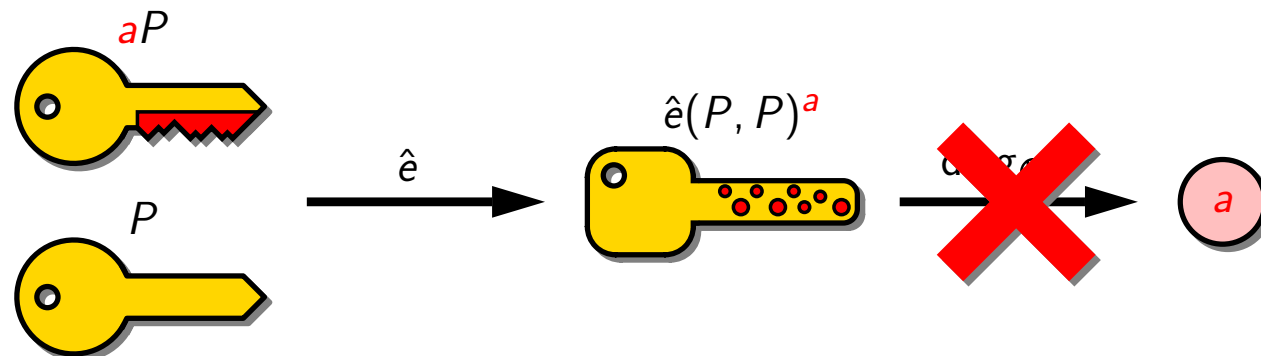
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- ▶ The discrete logarithm problem is usually easier in \mathbb{G}_2 than in \mathbb{G}_1
 - current security: $\sim 2^{80}$, equivalent to 80-bit symmetric encryption or RSA-1024
 - recommended security: $\sim 2^{128}$ (AES-128, RSA-3072)

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\times	$7 \lfloor \frac{m}{2} \rfloor + 29$	1121	$25 \lfloor \frac{m}{4} \rfloor + 93$	868
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- ▶ Software not well suited to small characteristic: need for hardware acceleration

Outline of the talk

- ▶ Pairing-based cryptography
- ▶ Pairings over elliptic curves
- ▶ **Finite-field arithmetic**
- ▶ Implementation results
- ▶ Concluding thoughts

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- ▶ Pairing-based cryptography
- ▶ Pairings over elliptic curves
- ▶ Finite-field arithmetic (only in characteristic 3)
- ▶ Implementation results
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Arithmetic over \mathbb{F}_{3^m}

- ▶ $f \in \mathbb{F}_3[x]$: degree- m irreducible polynomial over \mathbb{F}_3

$$f = x^m + f_{m-1}x^{m-1} + \dots + f_1x + f_0$$

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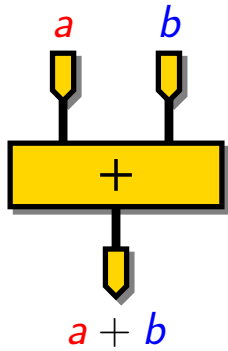
- ▶ $\mathbb{F}_{3^m} \cong \mathbb{F}_3[x]/(f)$

- ▶ $a \in \mathbb{F}_{3^m}$:

$$a = a_{m-1}x^{m-1} + \dots + a_1x + a_0$$

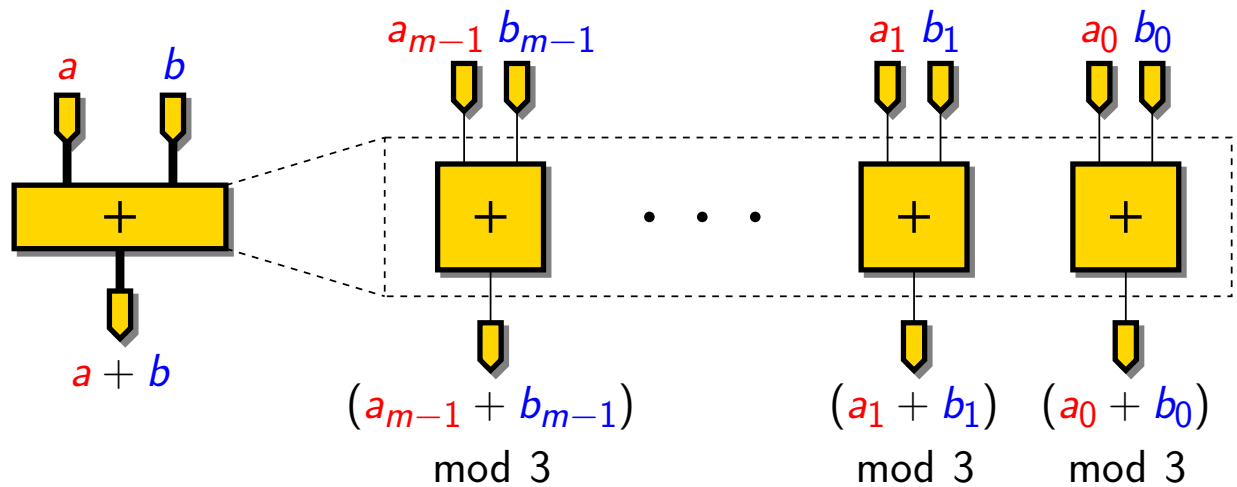
- ▶ Each element of \mathbb{F}_3 stored using two bits

Addition over \mathbb{F}_{3^m}



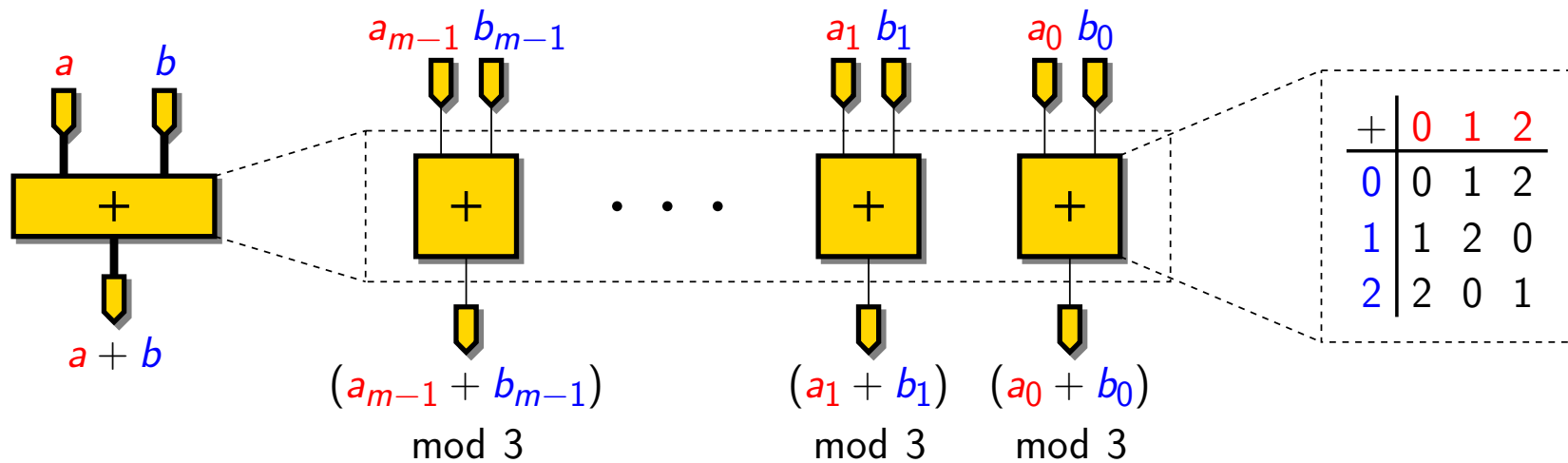
► $r = a + b = (a_{m-1} + b_{m-1})x^{m-1} + \dots + (a_1 + b_1)x + (a_0 + b_0)$

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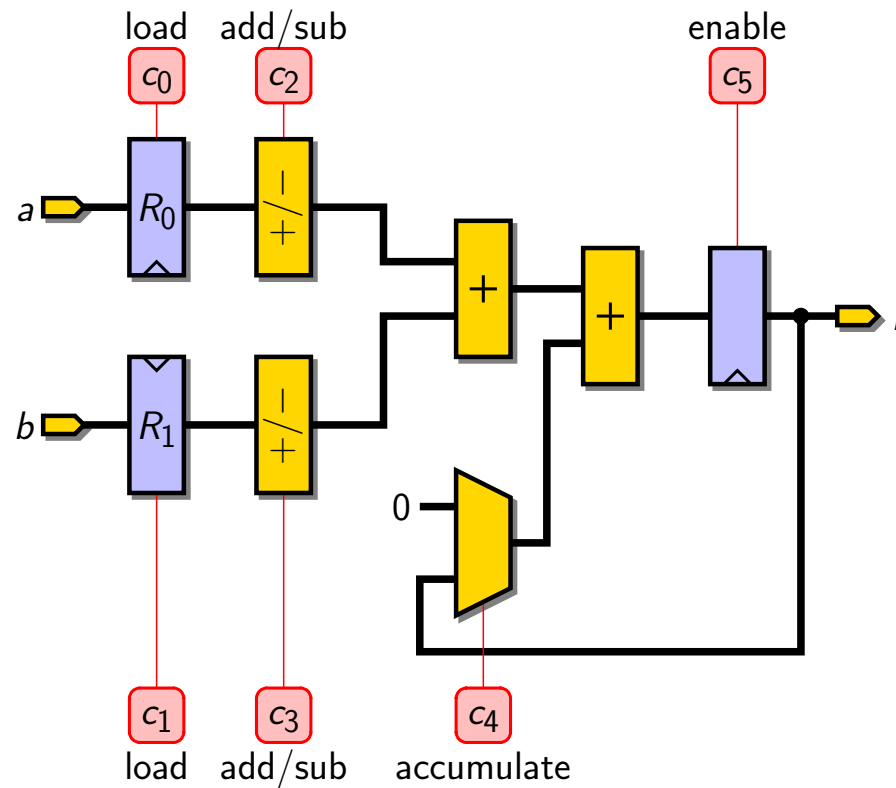
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- coefficient-wise additions over \mathbb{F}_3 : $r_i = (a_i + b_i) \bmod 3$
- addition over \mathbb{F}_3 : small look-up tables

Addition, subtraction and accumulation over \mathbb{F}_{3^m}



- sign selection: multiplication by 1 or 2

$$-a \equiv 2a \pmod{3}$$

- feedback loop for accumulation

Multiplication over \mathbb{F}_{3^m}

- ▶ Parallel-serial multiplication
 - multiplicand loaded in a parallel register
 - multiplier loaded in a shift register
- ▶ Most significant coefficients first (Horner scheme)
- ▶ D coefficients processed at each clock cycle: $\left\lceil \frac{m}{D} \right\rceil$ cycles per multiplication

Multiplication over \mathbb{F}_{3^m}

► Example for $D = 3$ (3 coefficients per iteration):

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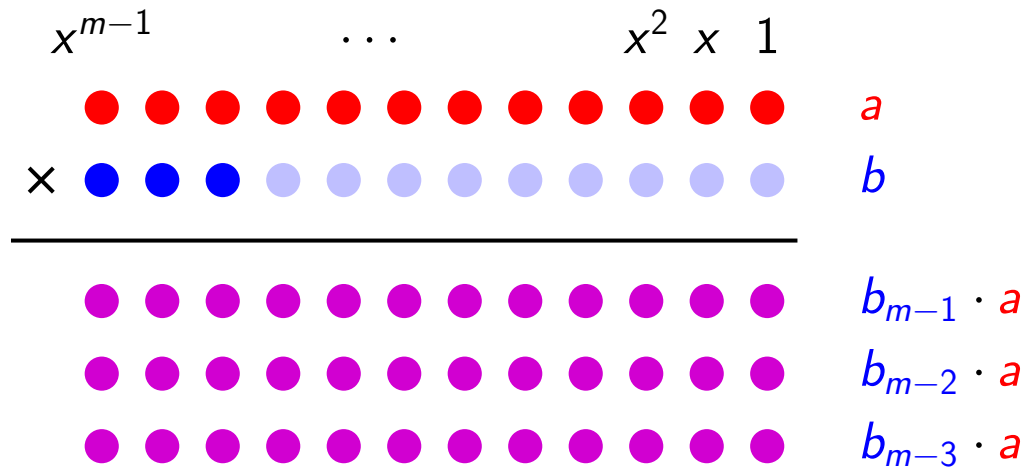
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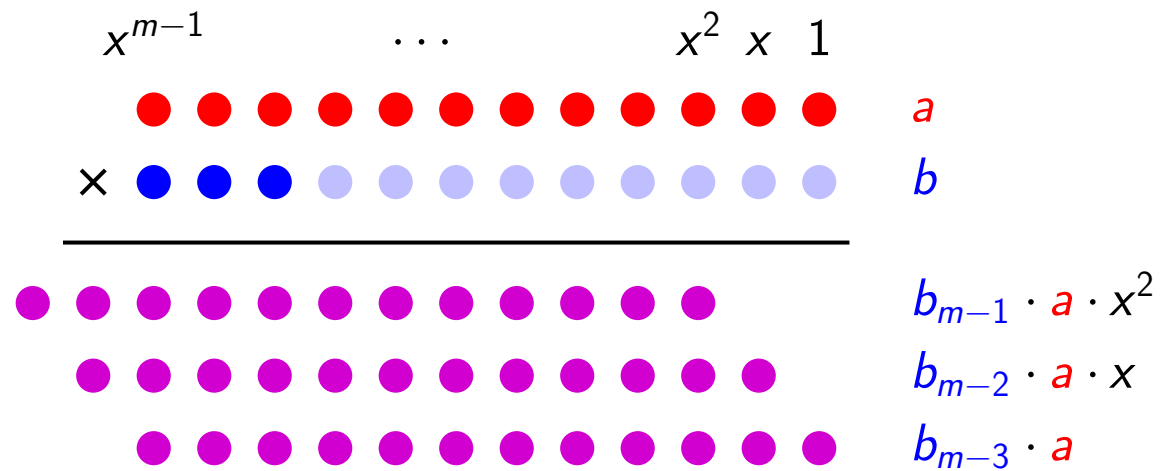
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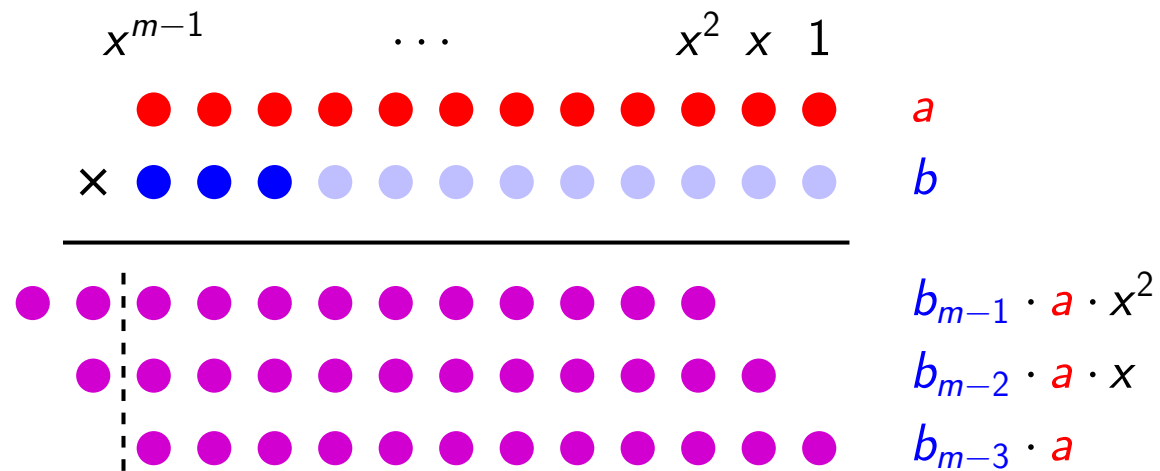
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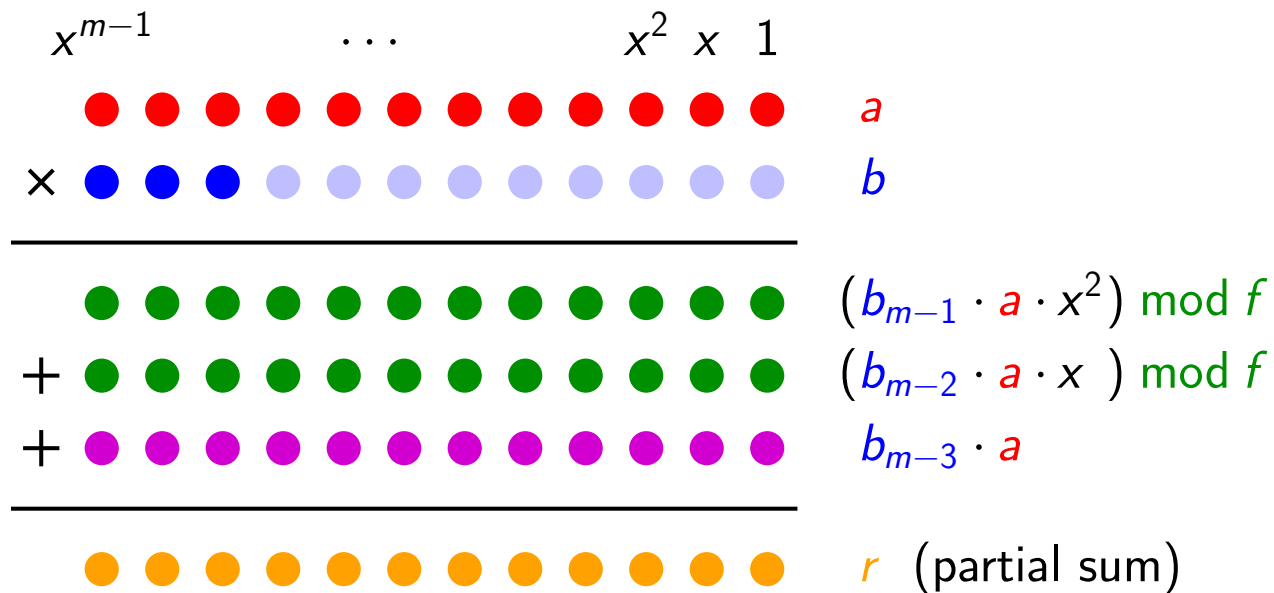
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 \hline
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 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & (b_{m-2} \cdot a \cdot x) \bmod f \\
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 \end{array}$$

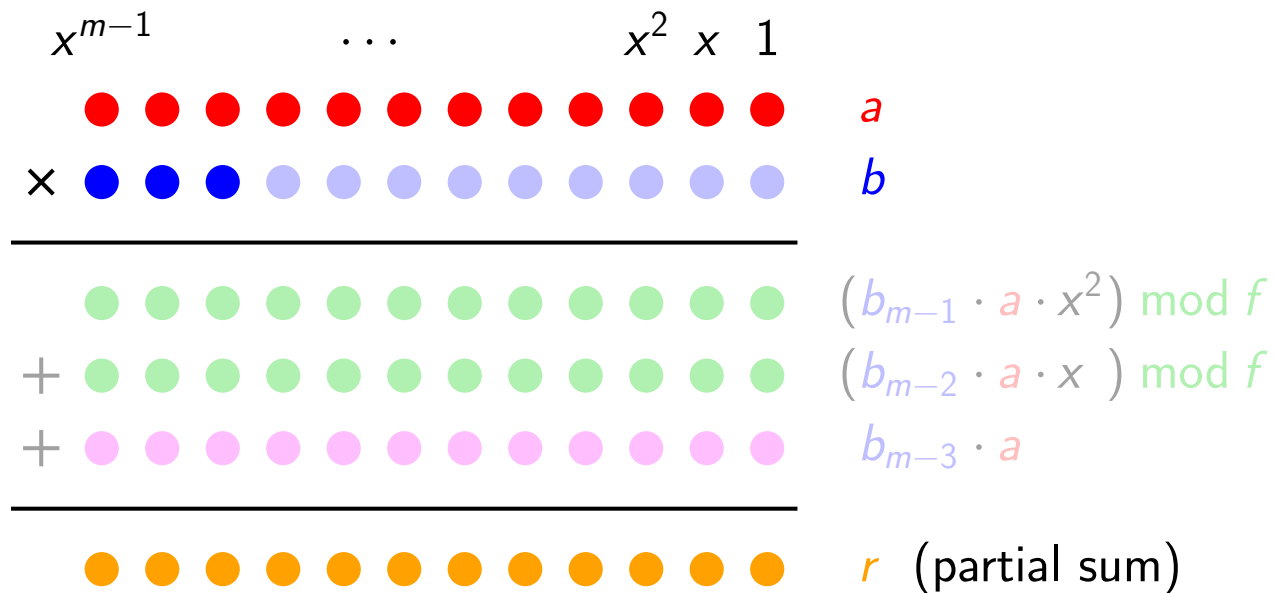
Multiplication over \mathbb{F}_{3^m}

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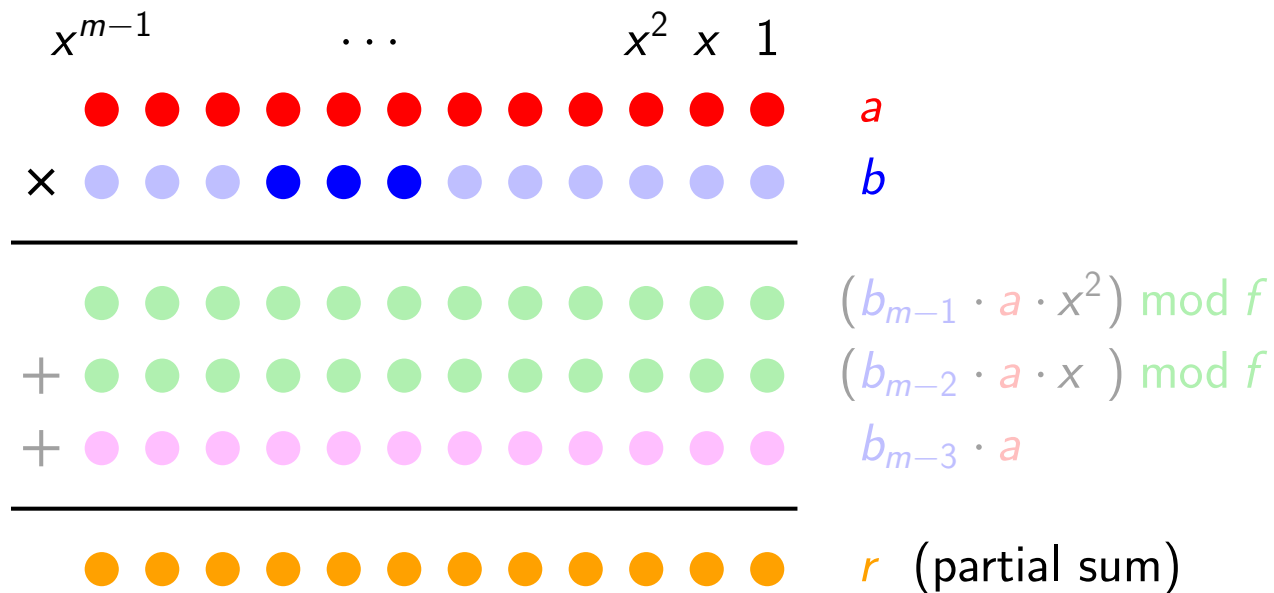
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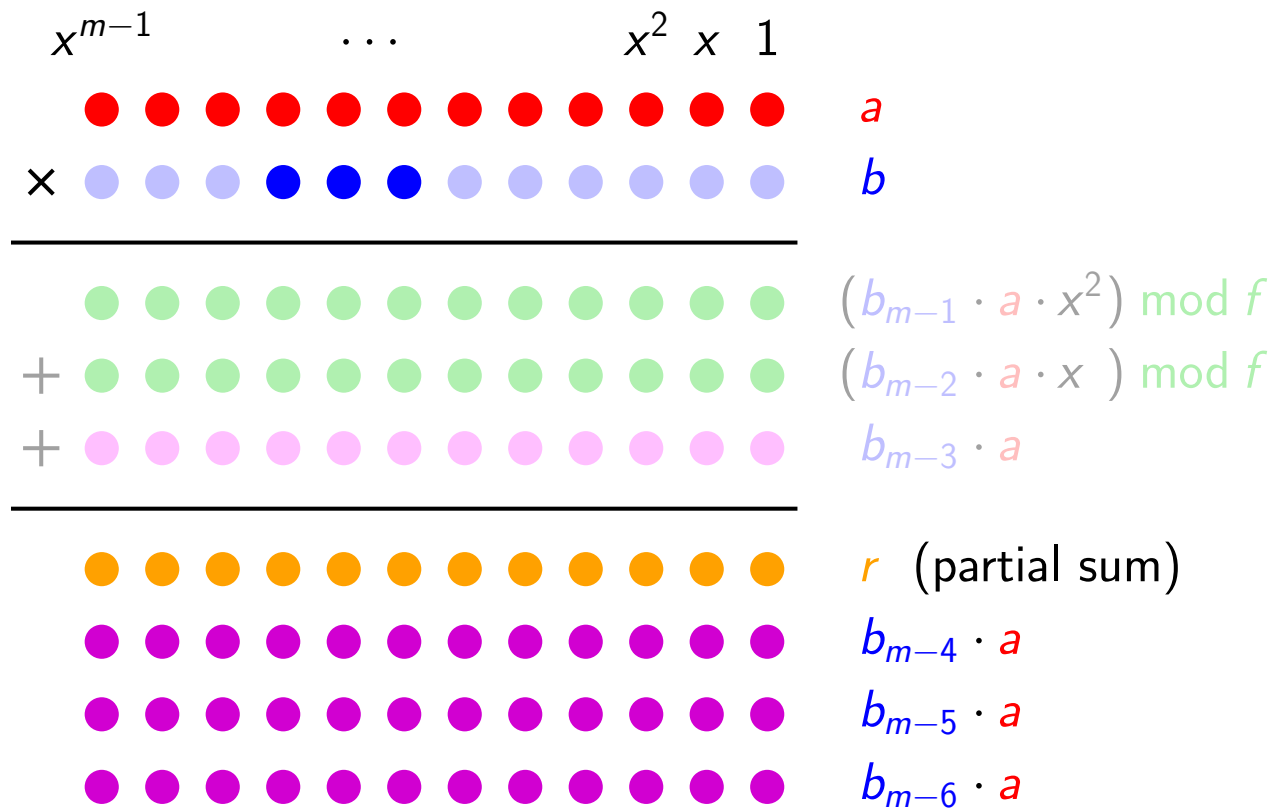
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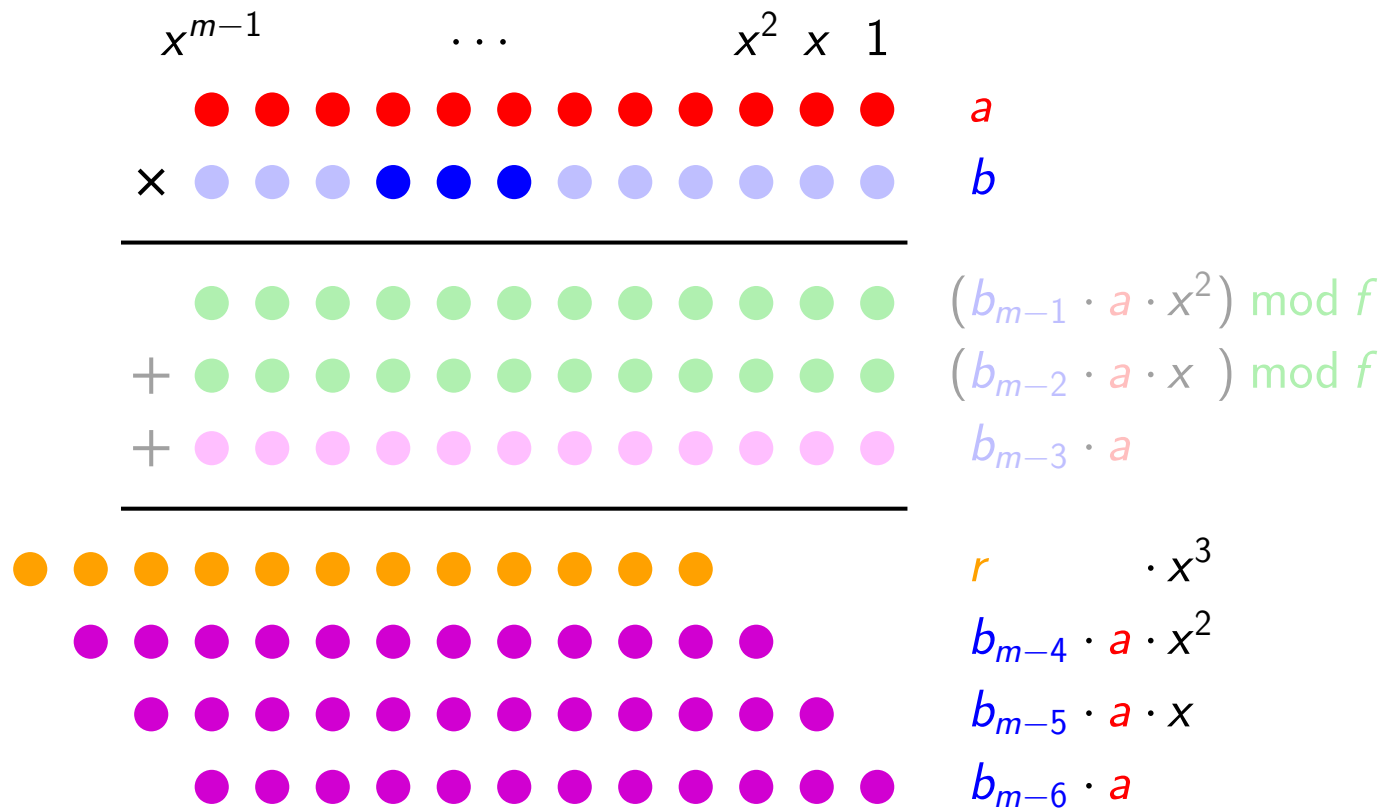
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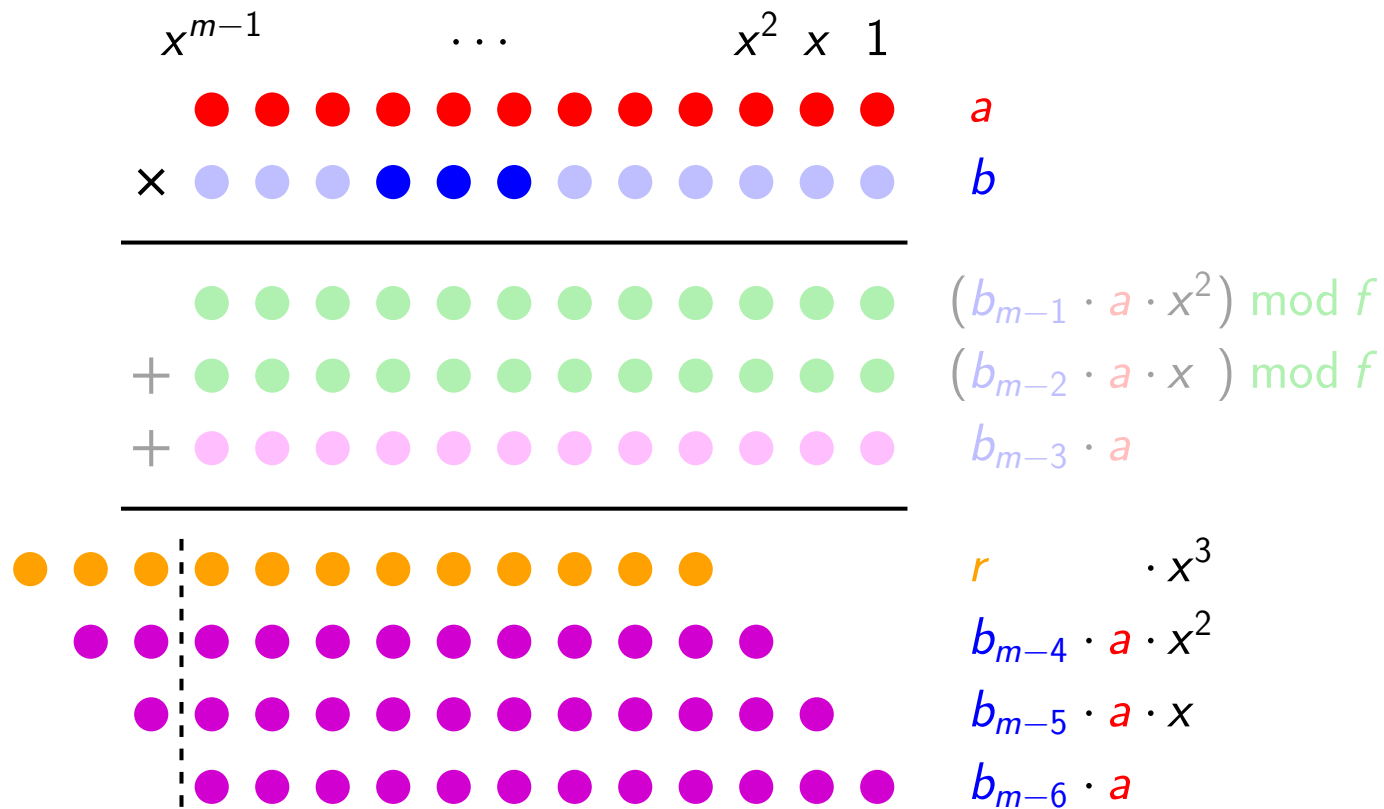
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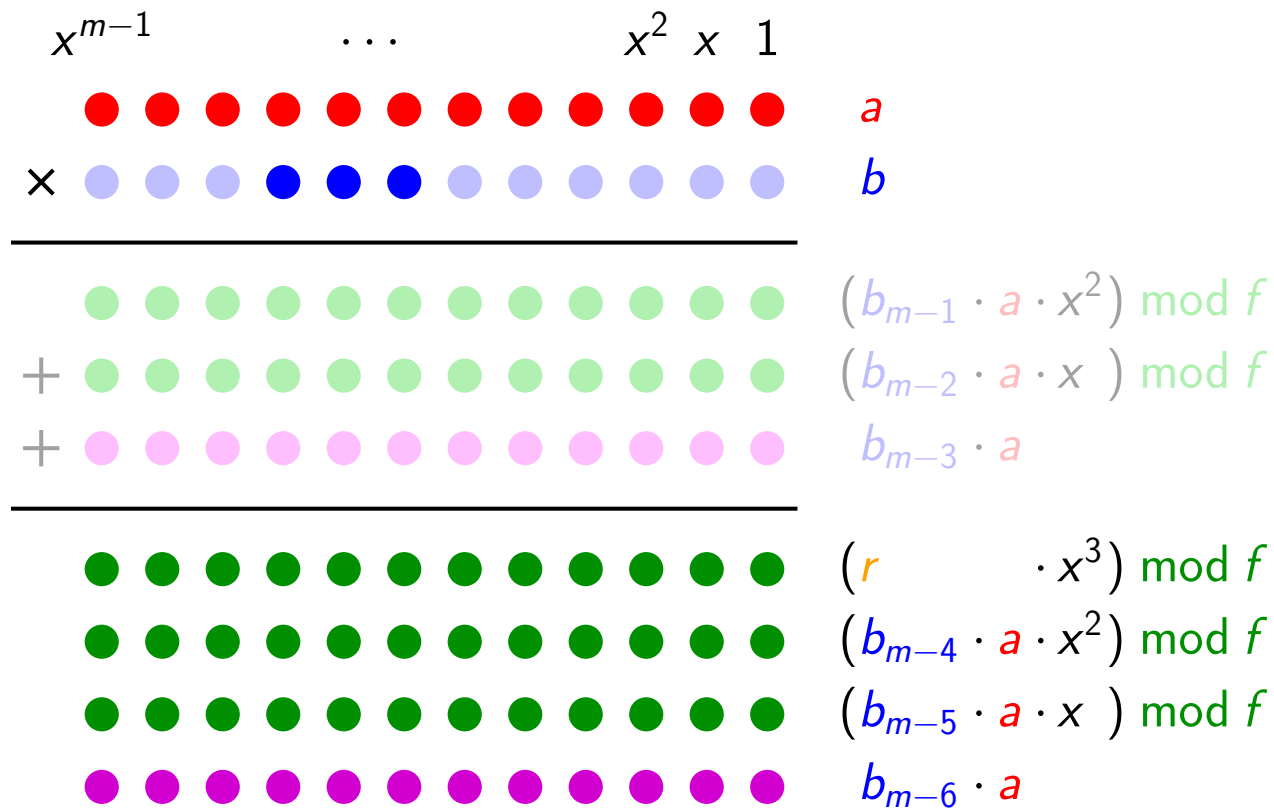
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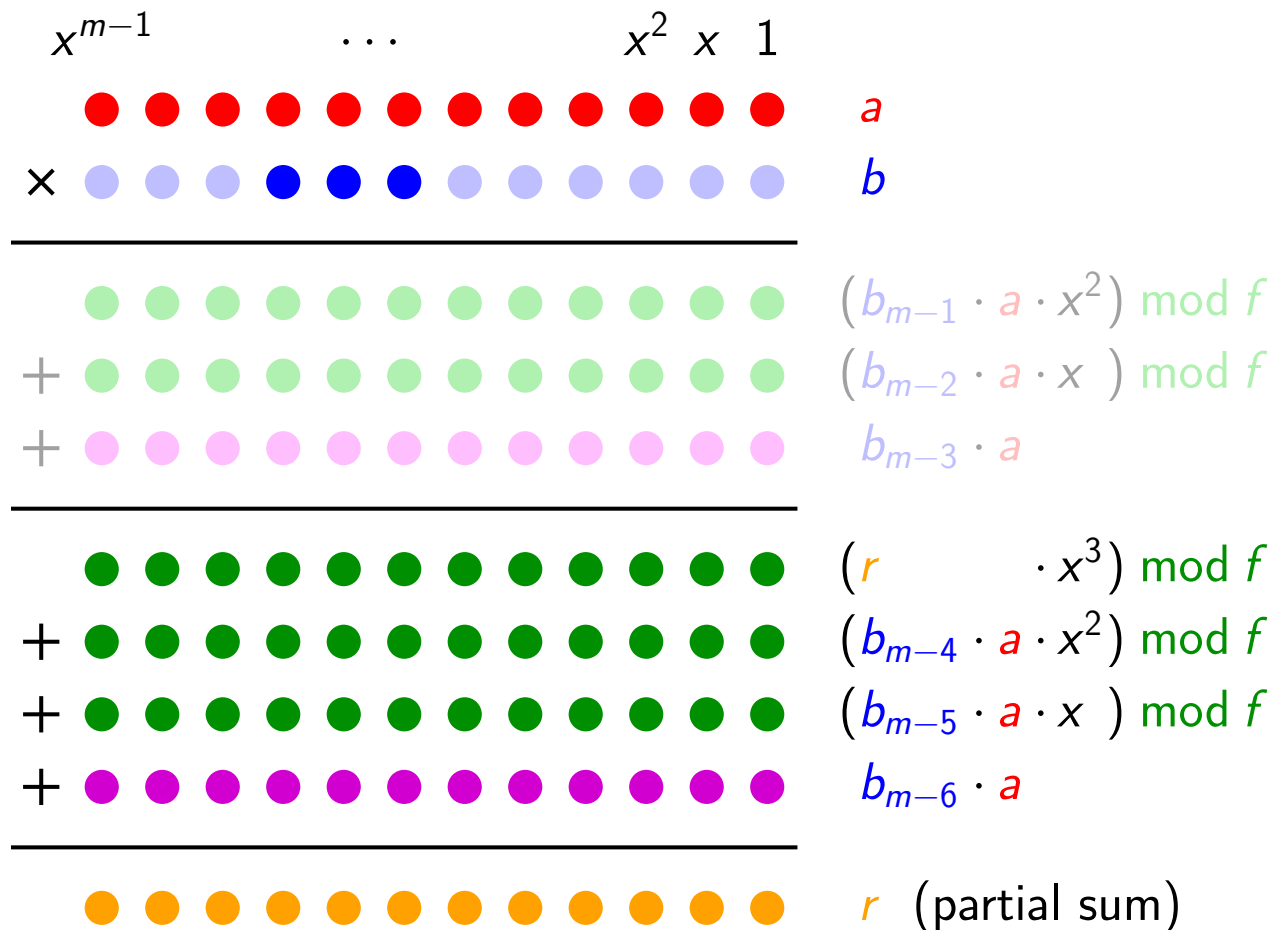
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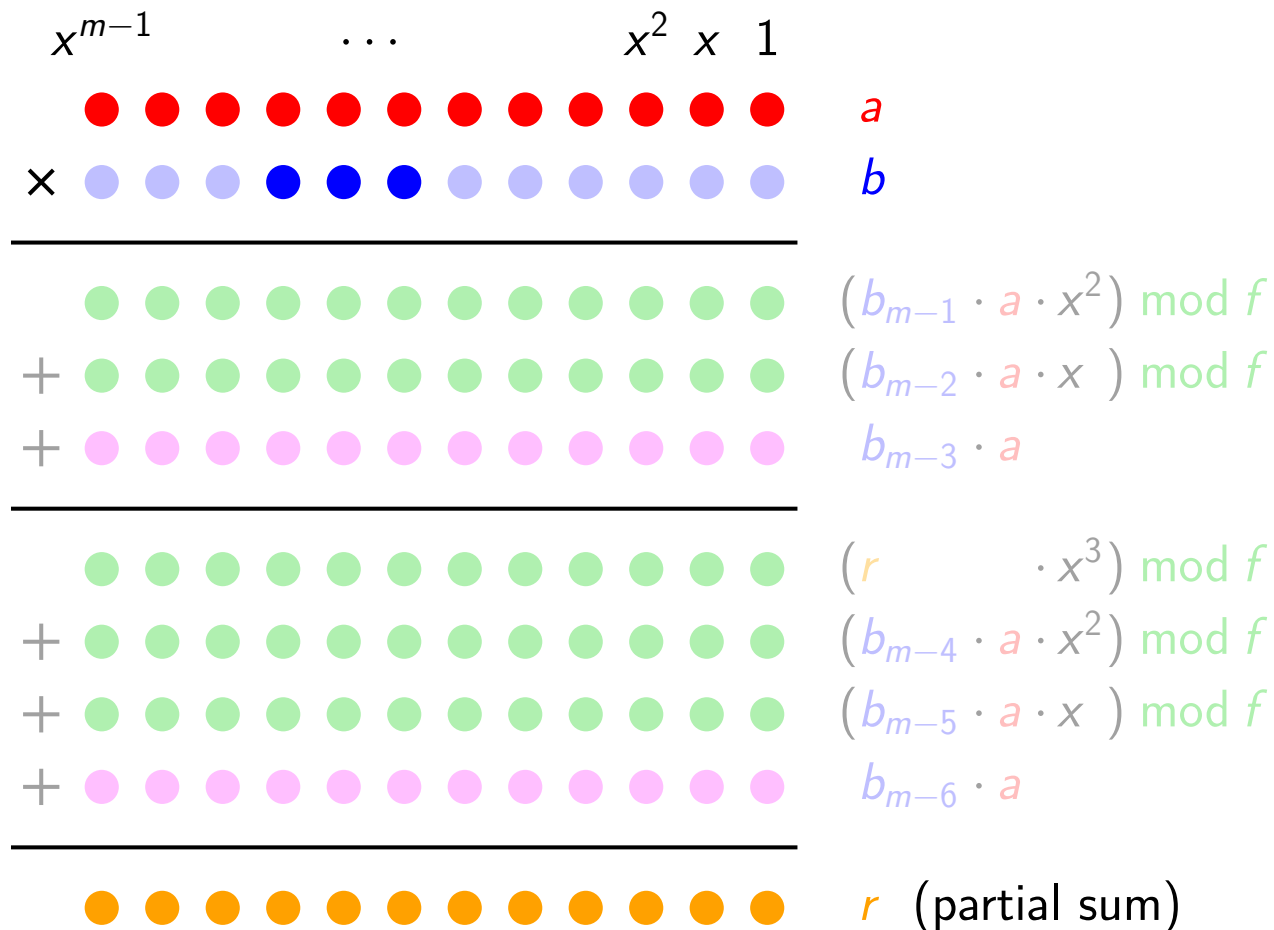
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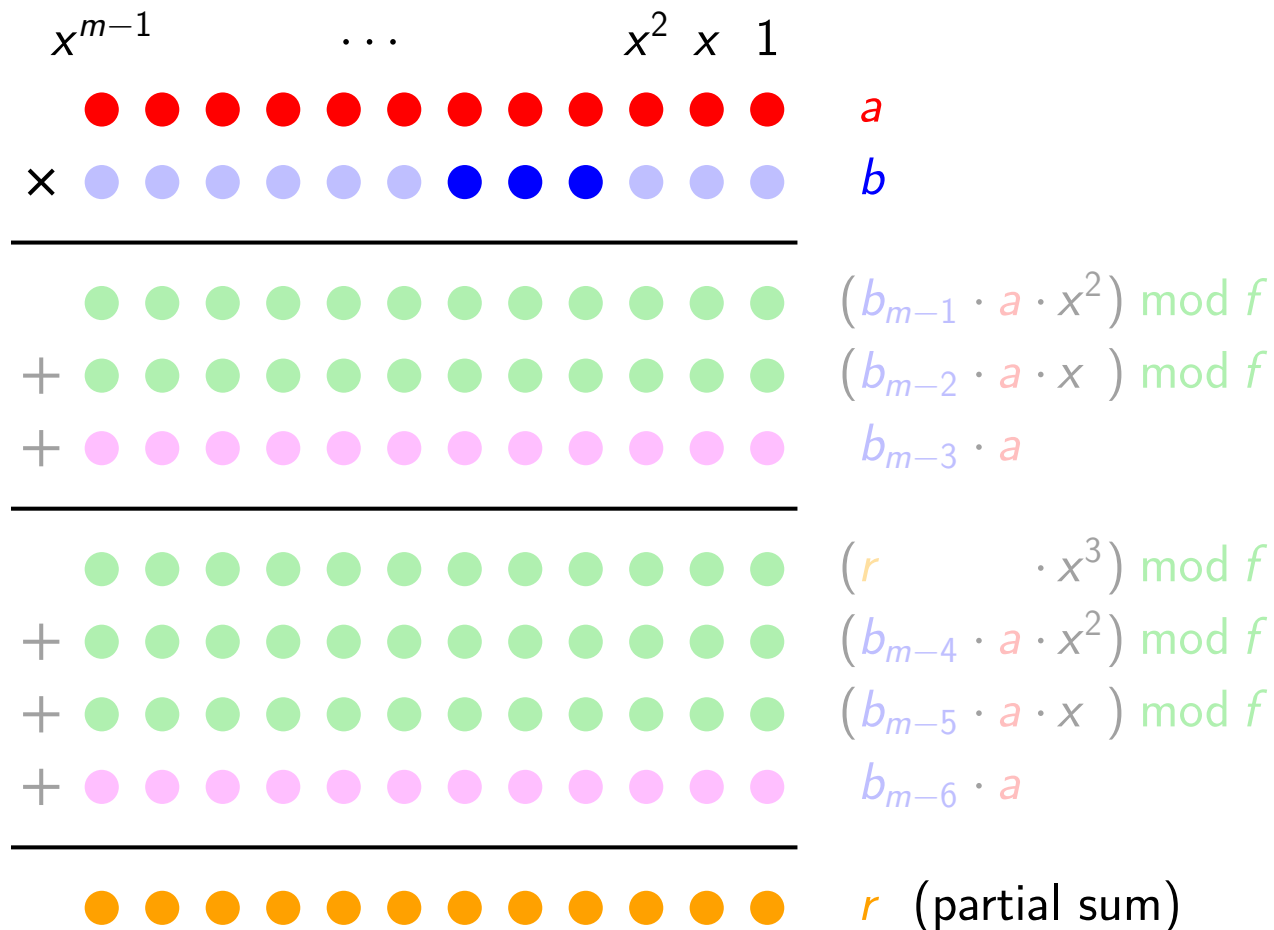
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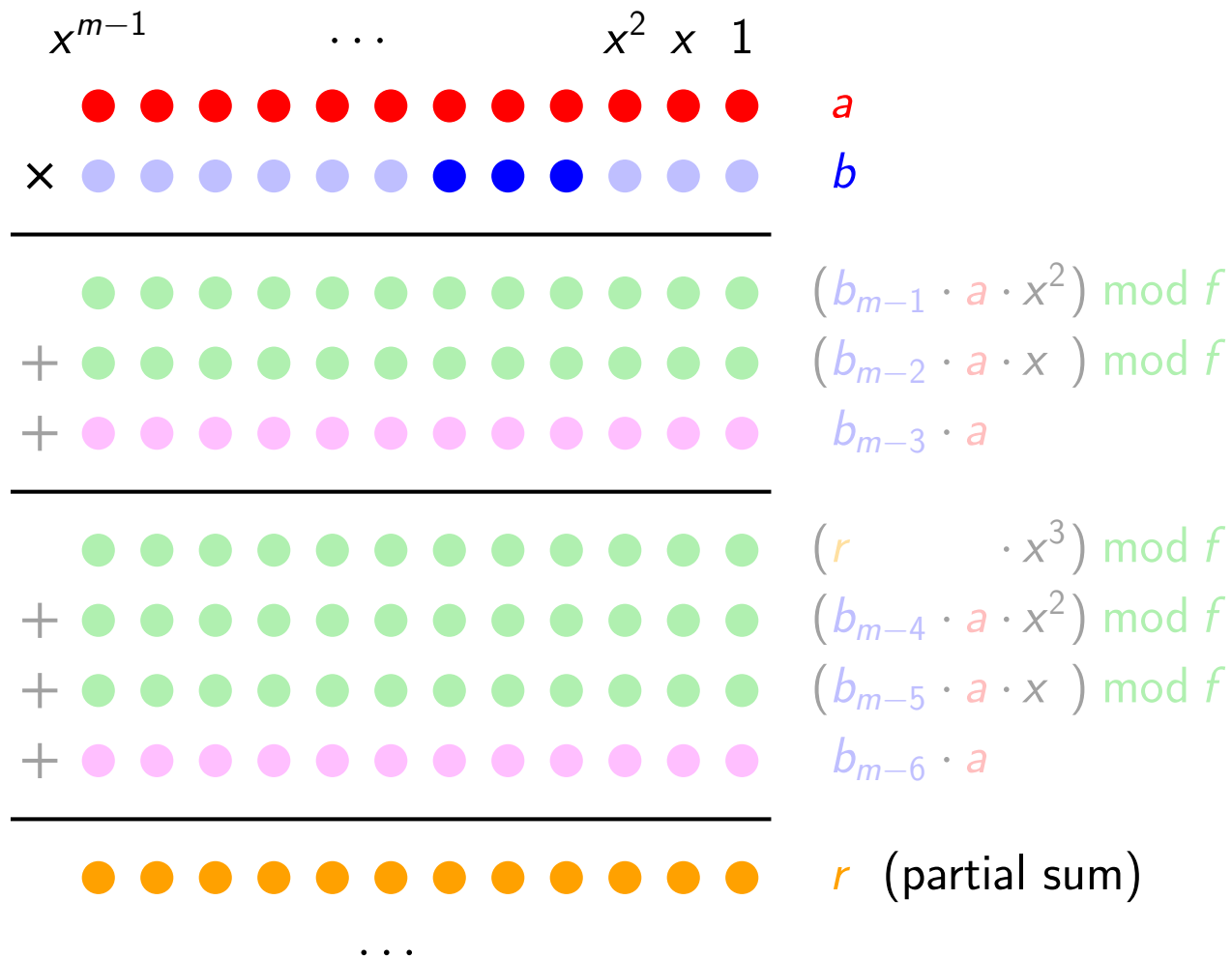
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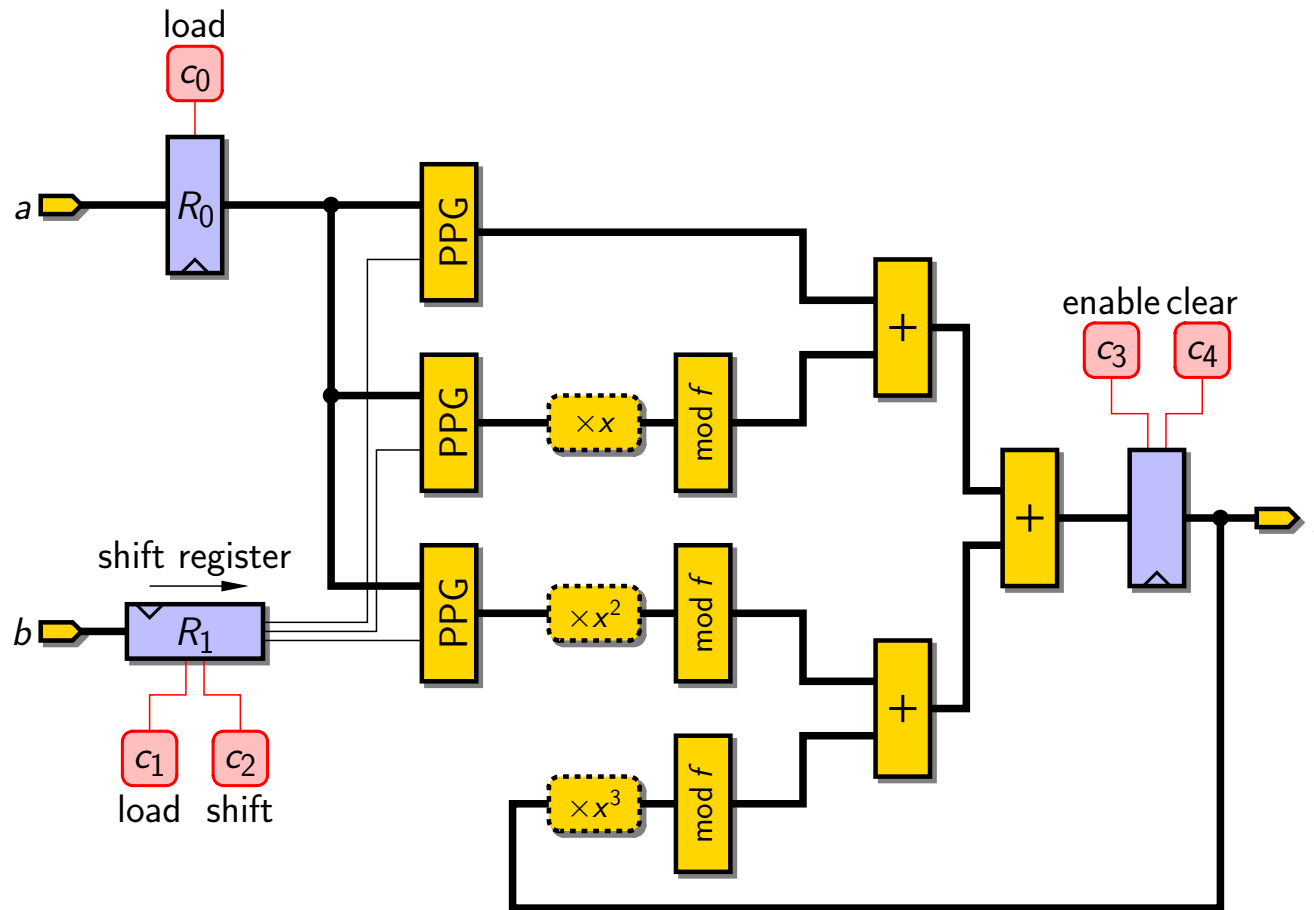
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 - $f = x^m + f_{m-1}x^{m-1} + \dots + f_1x + f_0$ gives
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Frobenius map over \mathbb{F}_{3^m} : cubing

▶ Since $\binom{3}{1} = \binom{3}{2} = 3$:

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- ▶ Symbolic computation of the reduction:
each coefficient of the result is a linear combination of the a_i 's

$$a^3 \bmod f = \sum_{j=0}^{n-1} w_j \cdot \mu_j$$

with $w_j \in \mathbb{F}_3$, $\mu_j \in \mathbb{F}_{3^m}$, and $\mu_{j,i} \in \{0\} \cup \{a_{m-1}, \dots, a_1, a_0\}$

Frobenius map over \mathbb{F}_{3^m}

► Example for $m = 97$ and $f = x^{97} + x^{12} + 2$:

$$\begin{aligned}
 a^3 \bmod f &= (a_{32}x^{96} + a_{64}x^{95} + a_{96}x^{94} + \dots + a_{33}x^2 + a_{65}x + a_0) \times 1 \\
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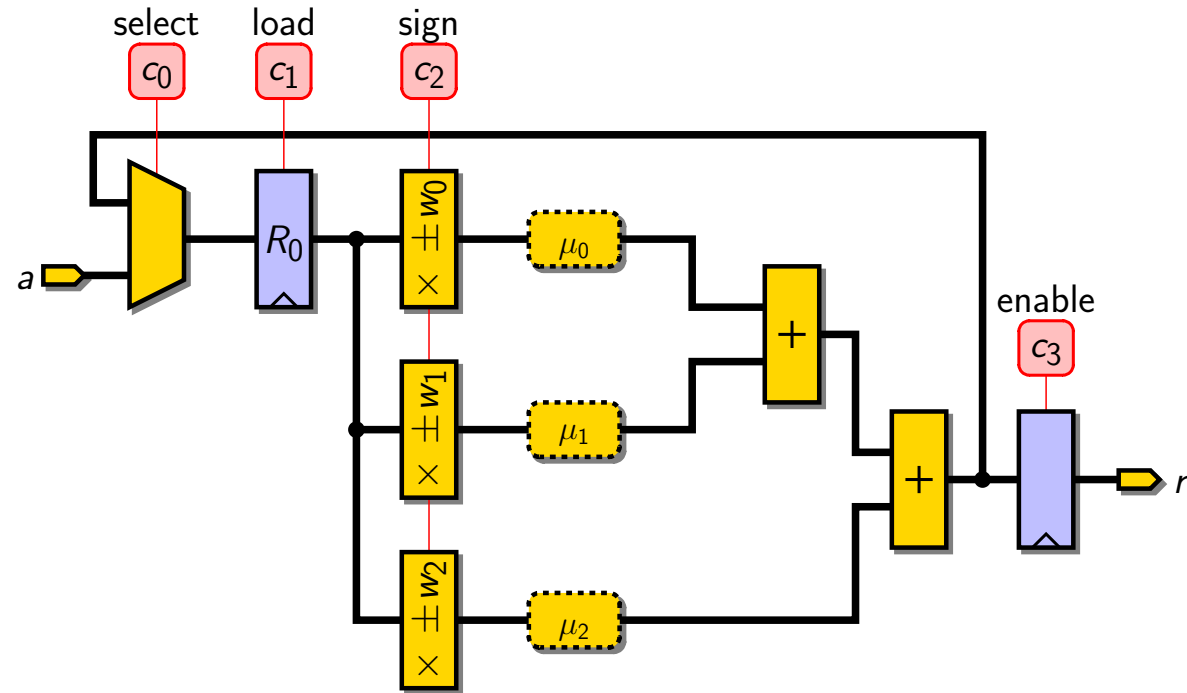
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 \end{aligned}$$

► Required hardware:

- only wires to compute the μ_j 's
- multiplications over \mathbb{F}_3 for the weights w_j
- multi-operand addition over \mathbb{F}_{3^m}

Frobenius map over \mathbb{F}_{3^m}



- feedback loop for successive cubings
- sign selection for computing either a^3 or $-a^3$

Inversion over \mathbb{F}_{3^m}

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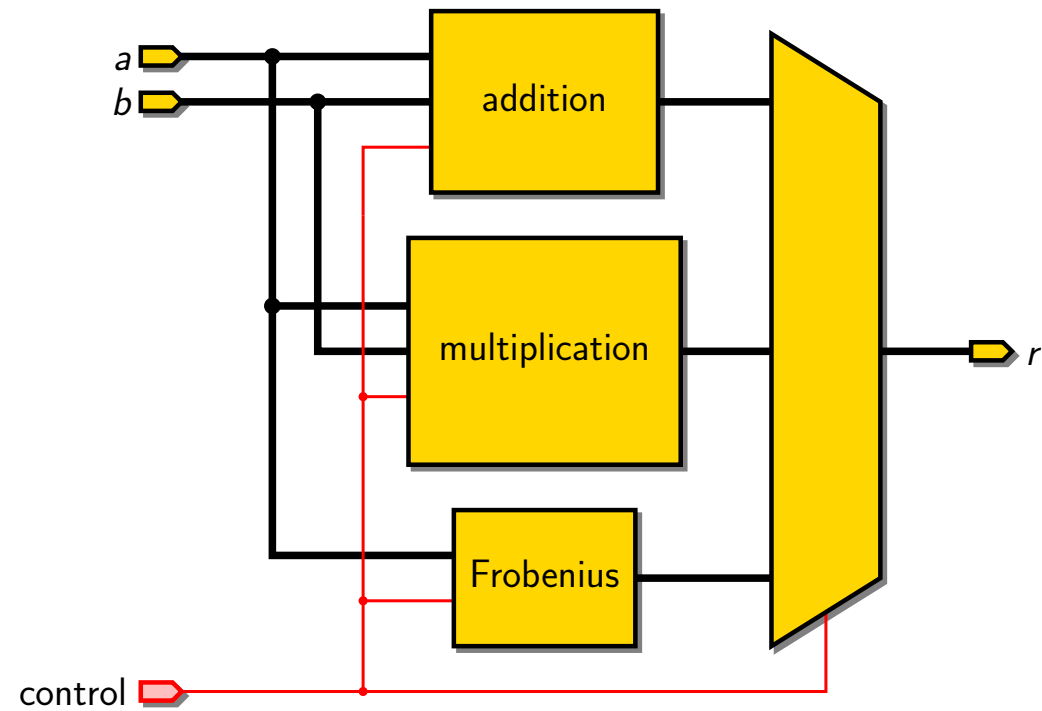
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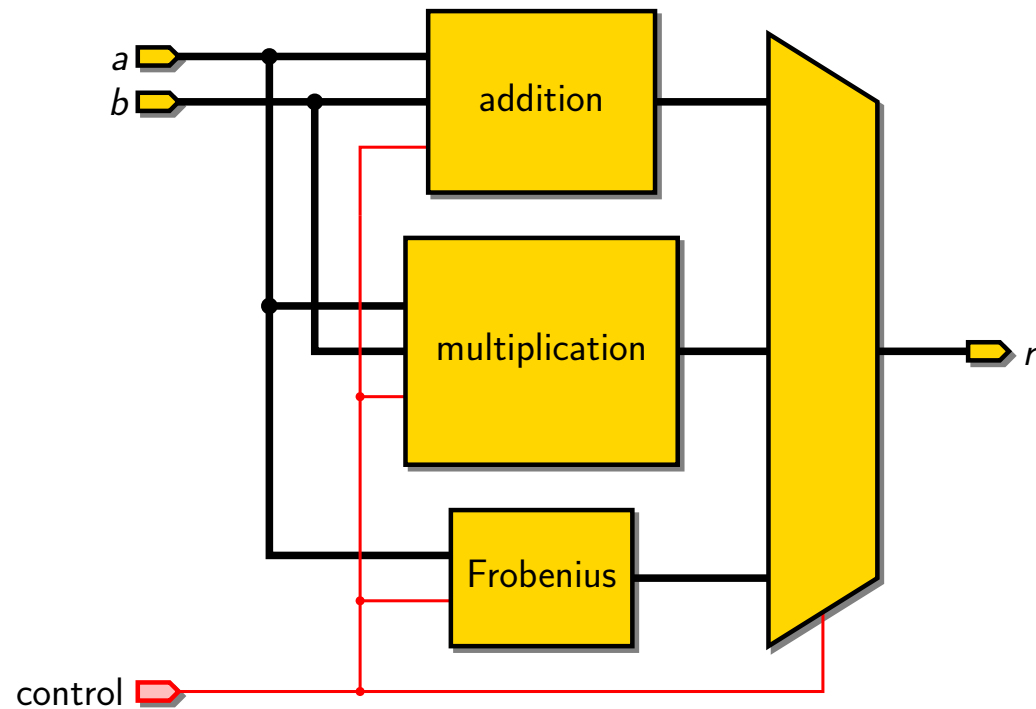
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- requires only multiplications and cubings over \mathbb{F}_{3^m}
- only one inversion for the full pairing: delay overhead is negligible ($< 1\%$)

The full processing element



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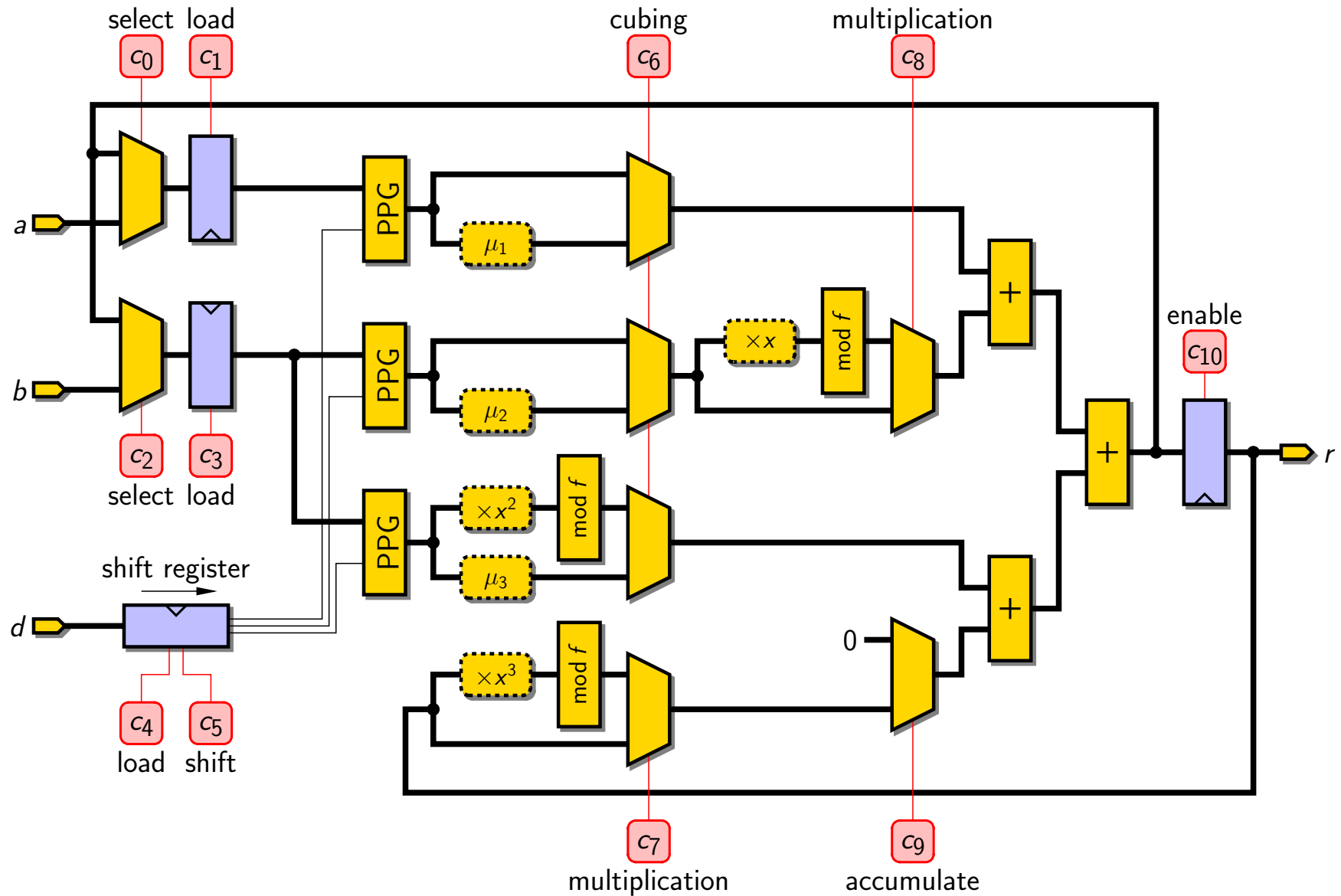


- ▶ For the Tate pairing:
limited parallelism between additions, multiplications and Frobenius maps
- ▶ Can we share hardware resources between the three operators?

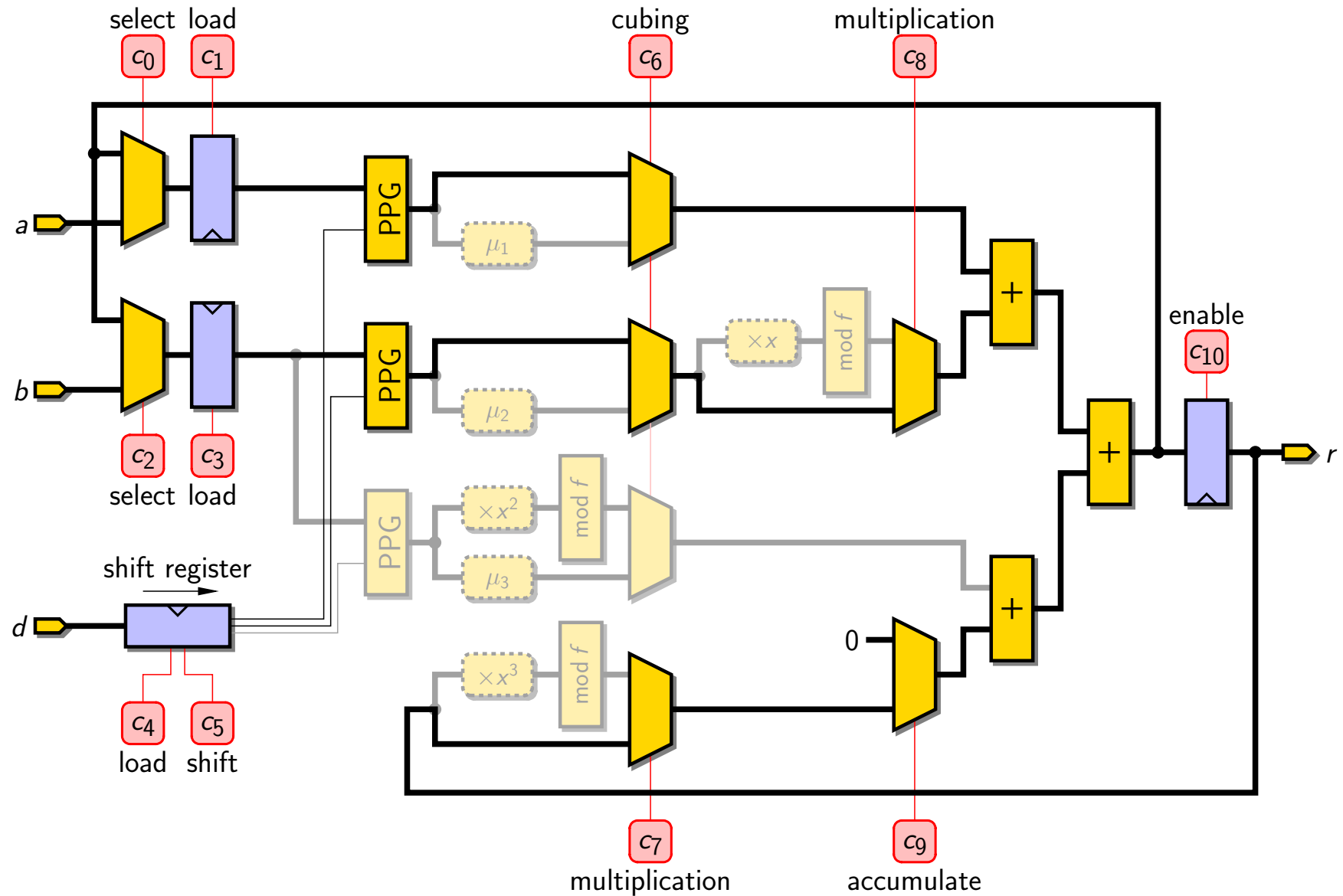
What can we share?

- ▶ Input and output registers
- ▶ Partial product generators:
 - sign selection for the addition / subtraction
 - partial products for the multiplication
 - multiplication by the w_j 's for the Frobenius map
- ▶ Multi-operand addition tree
- ▶ Feedback loops for accumulation

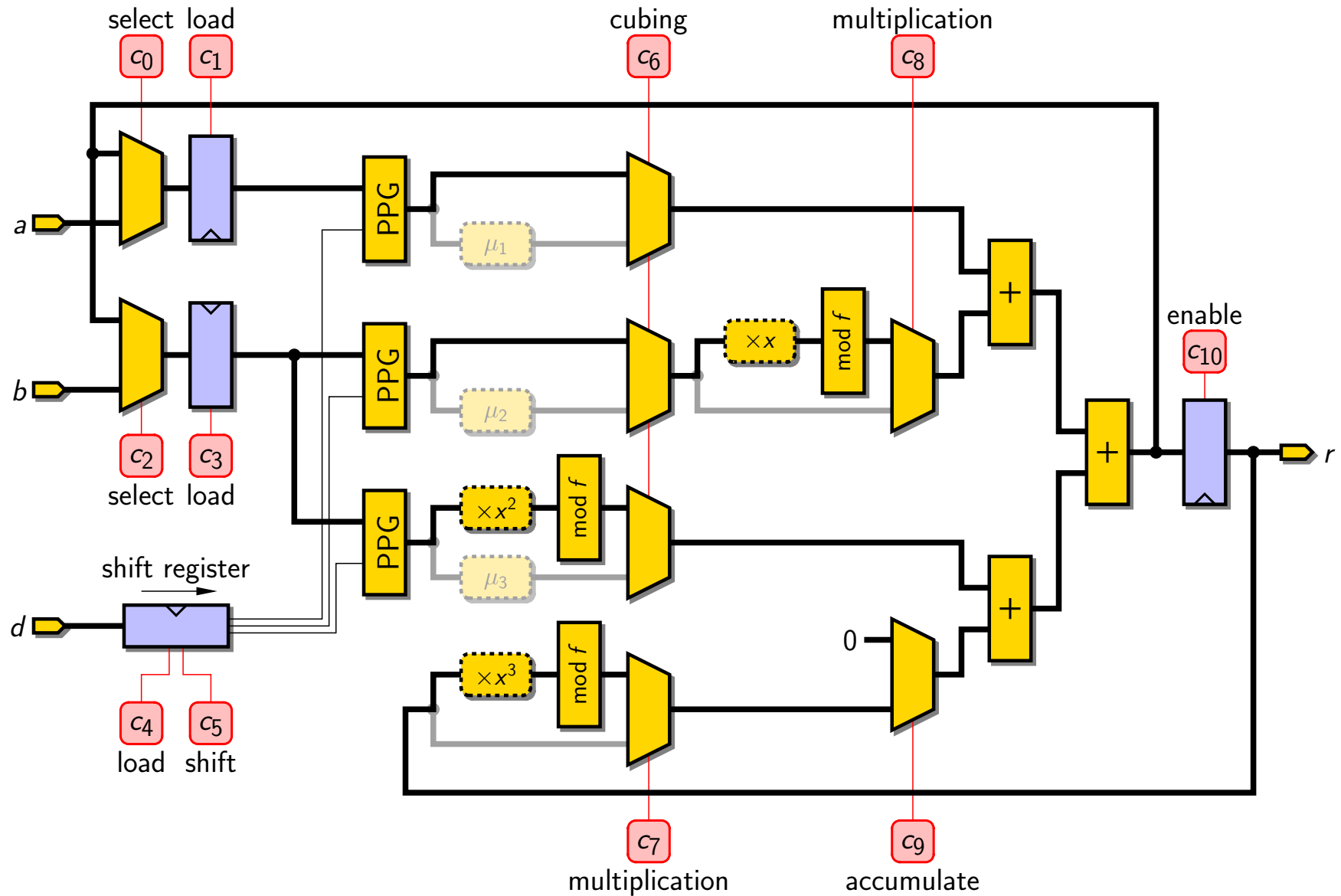
Our unified operator



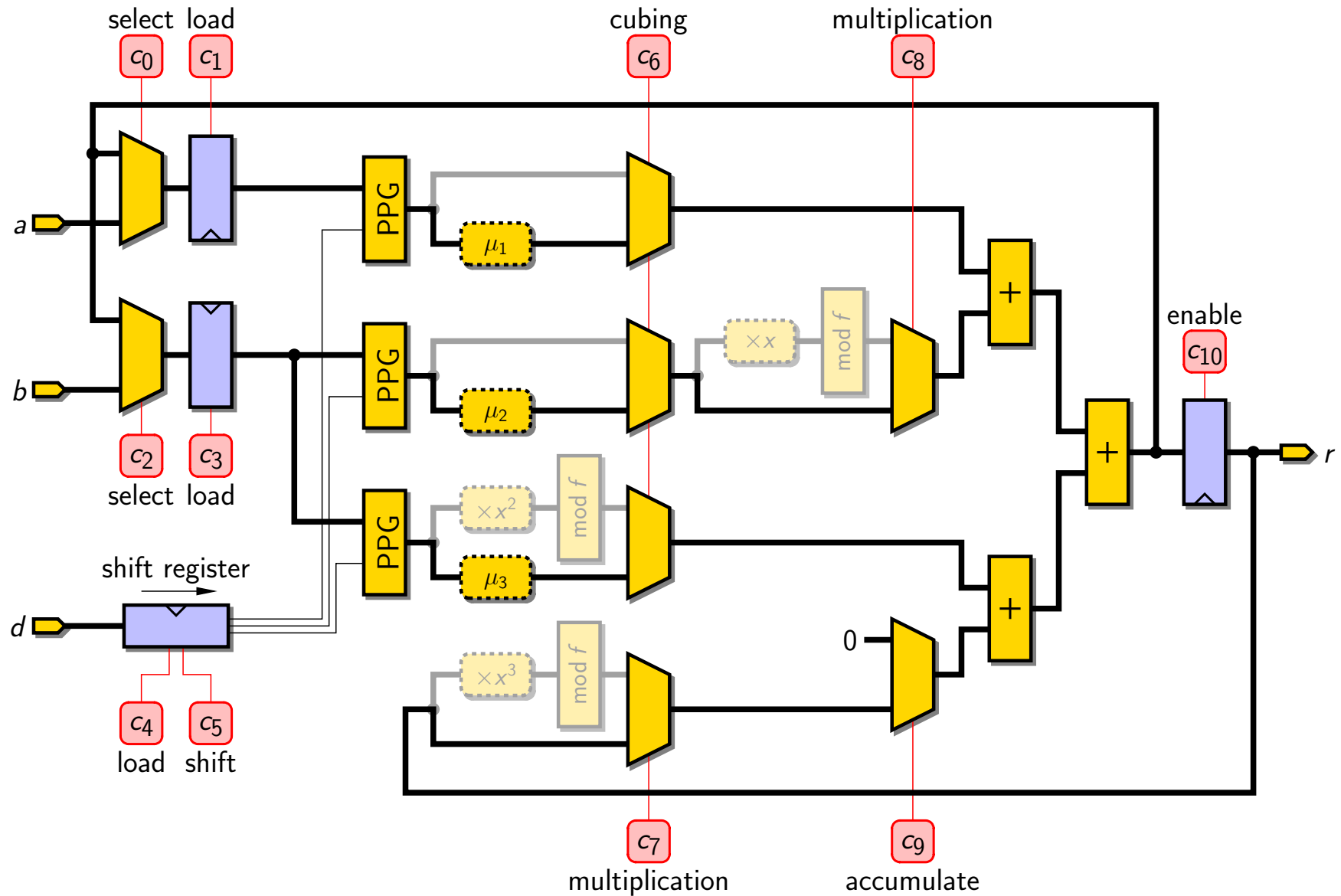
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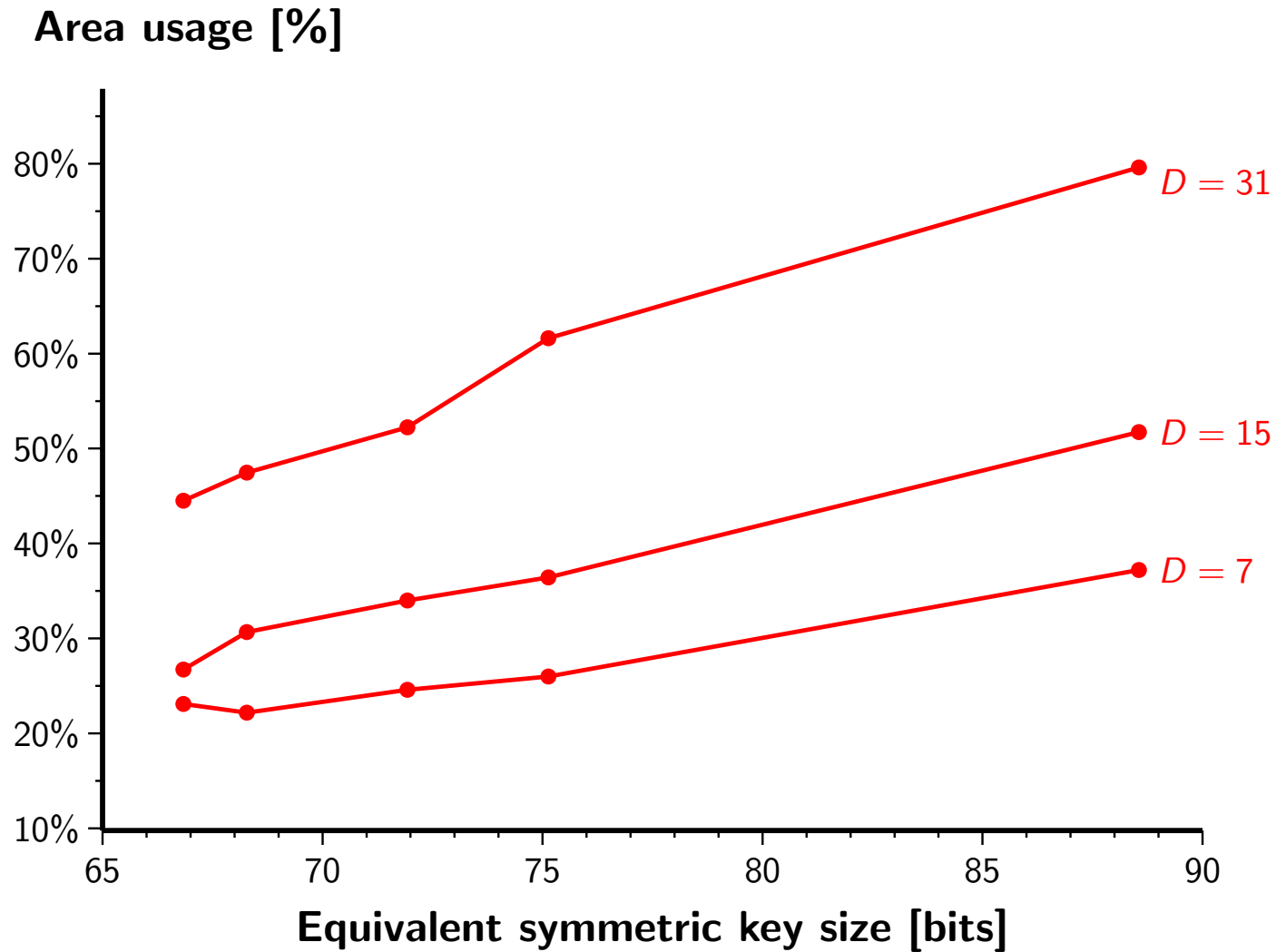
Outline of the talk

- ▶ Pairing-based cryptography
- ▶ Pairings over elliptic curves
- ▶ Finite-field arithmetic
- ▶ **Implementation results**
- ▶ Concluding thoughts

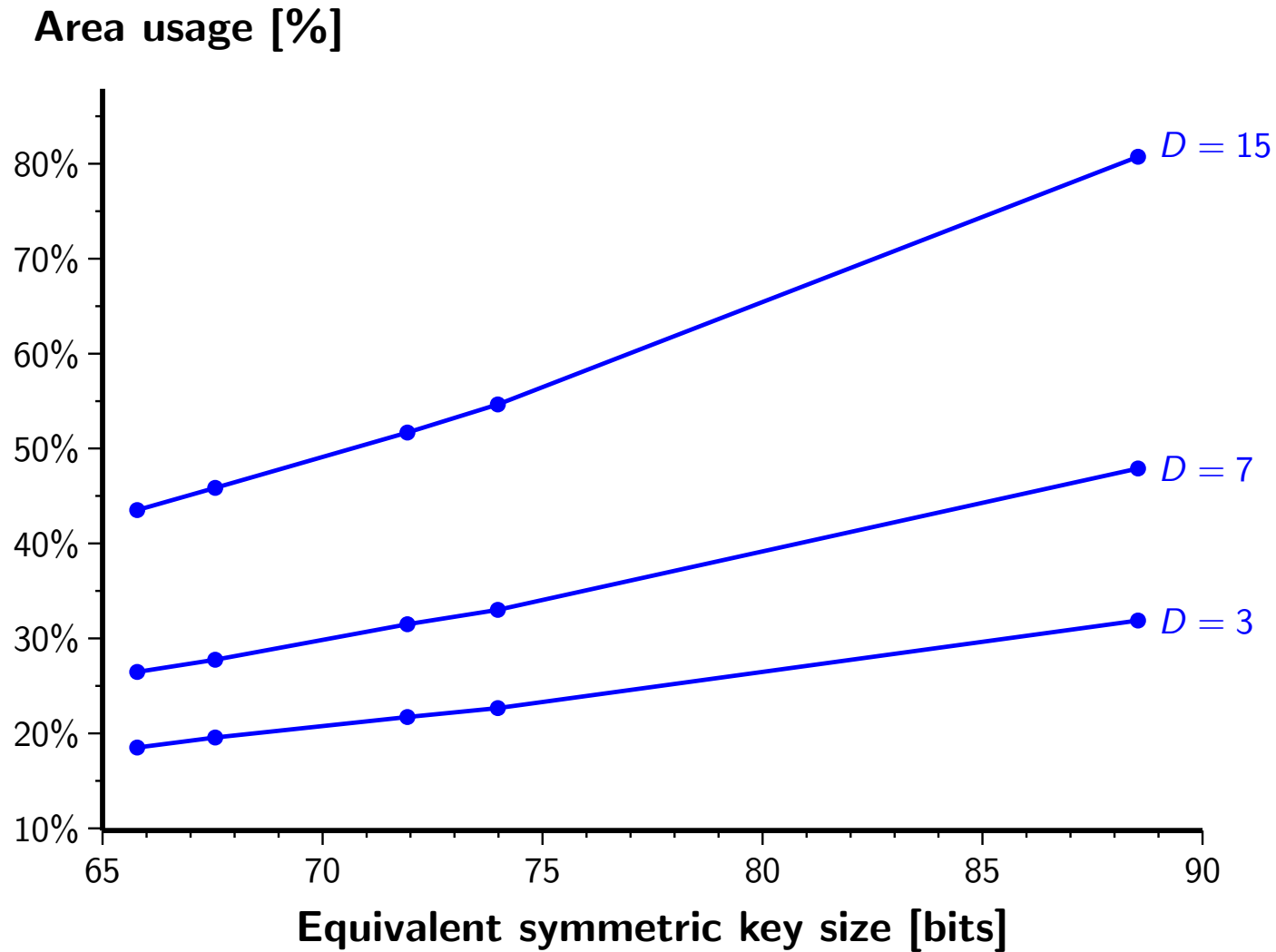
Experimental setup

- ▶ Full coprocessor for computation of the Tate pairing
- ▶ Architecture based on our unified operator
- ▶ Prototyped on a Xilinx Virtex-II Pro 20 FPGA (mid-range model)
- ▶ Post place-and-route results: area, computation time, AT product

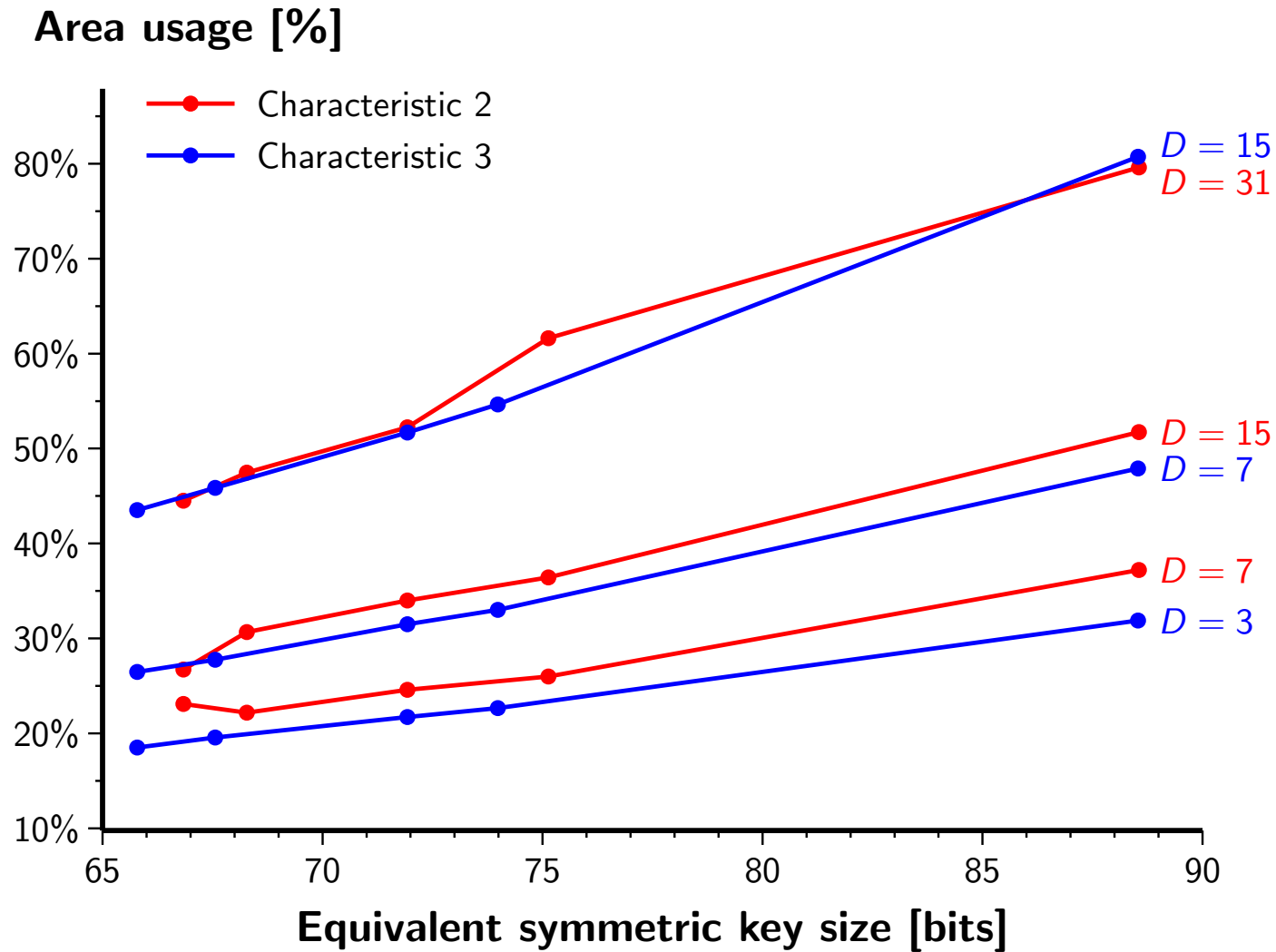
Coprocessor area (characteristic 2)



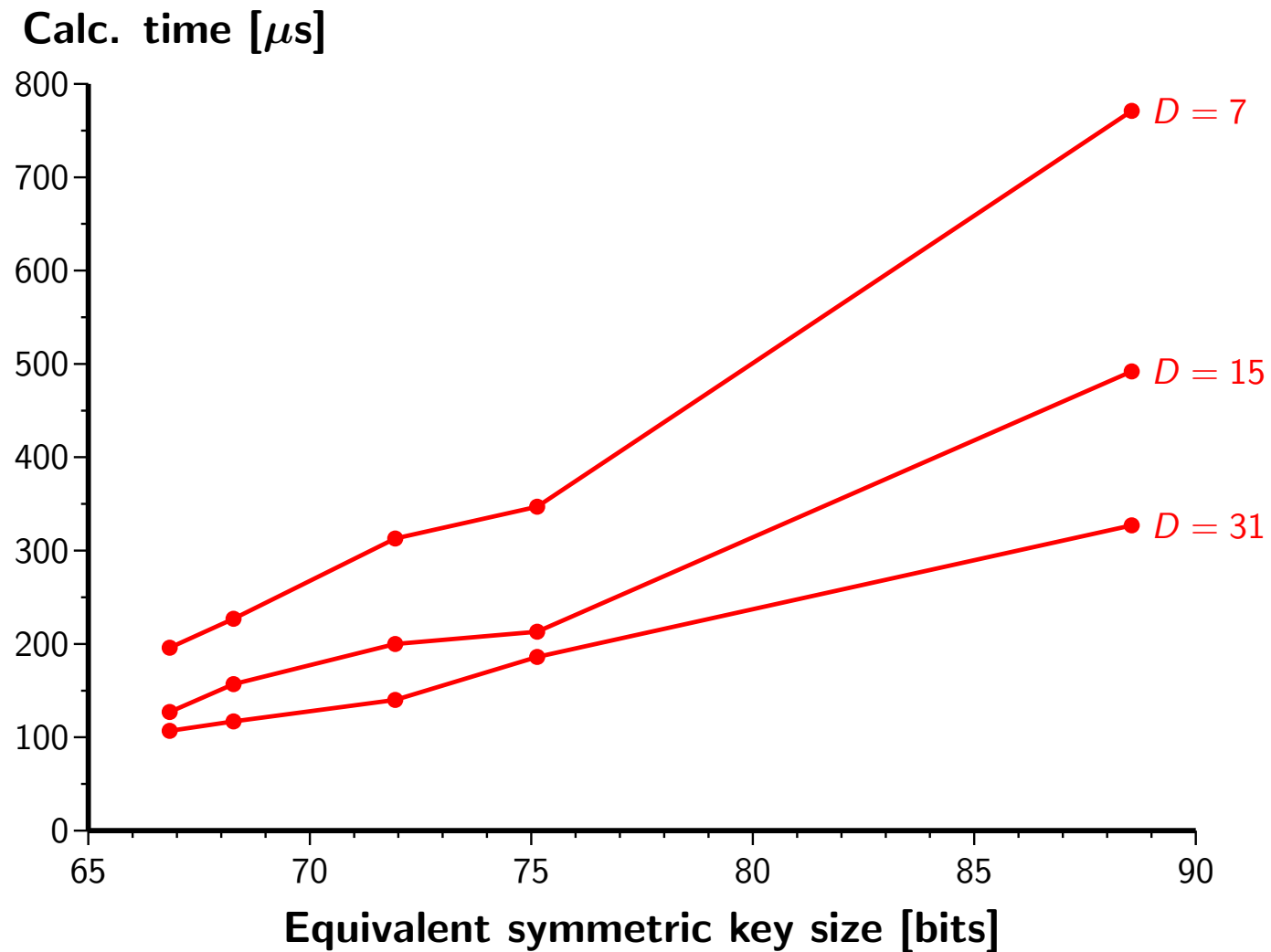
Coprocessor area (characteristic 3)



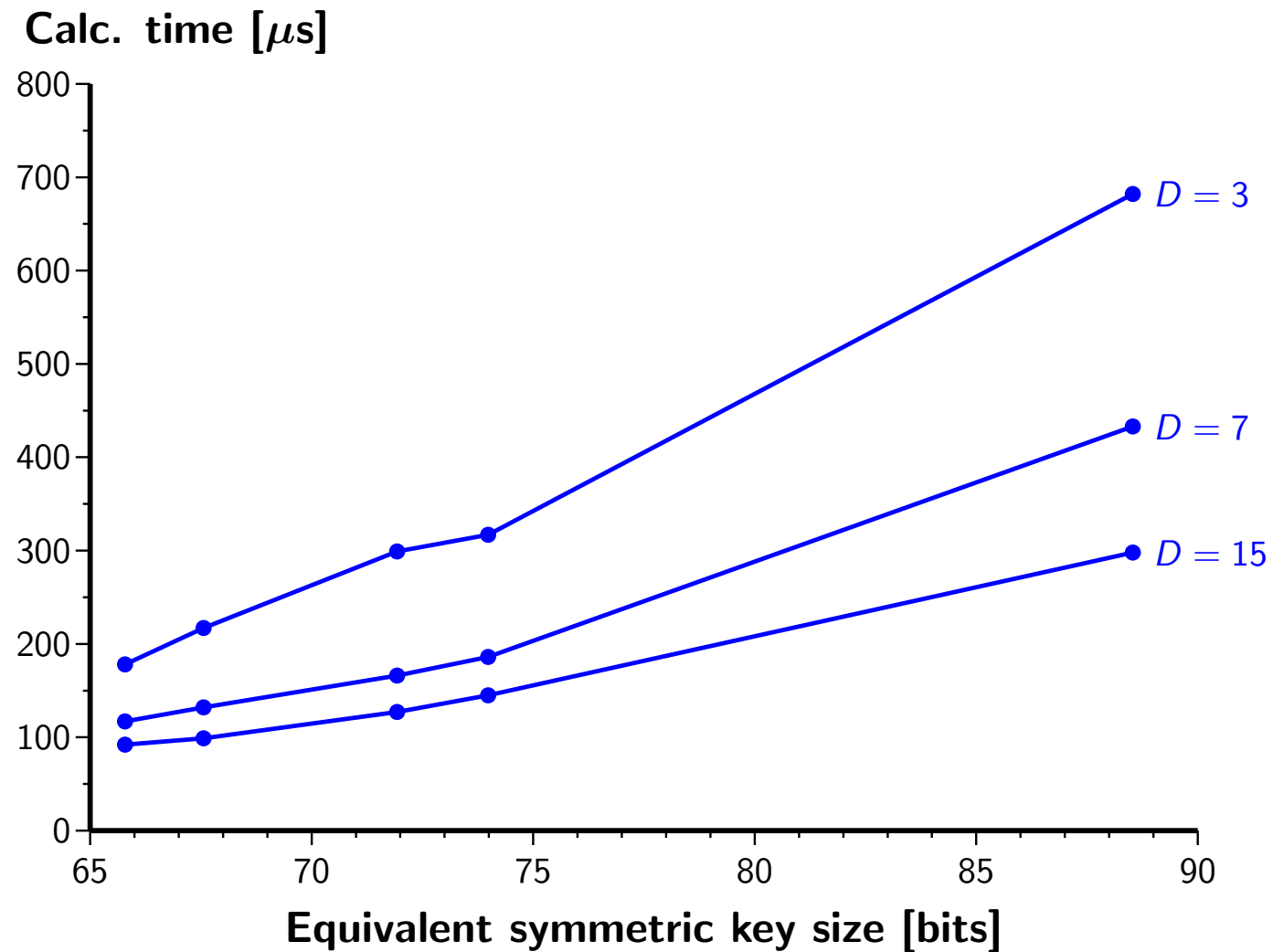
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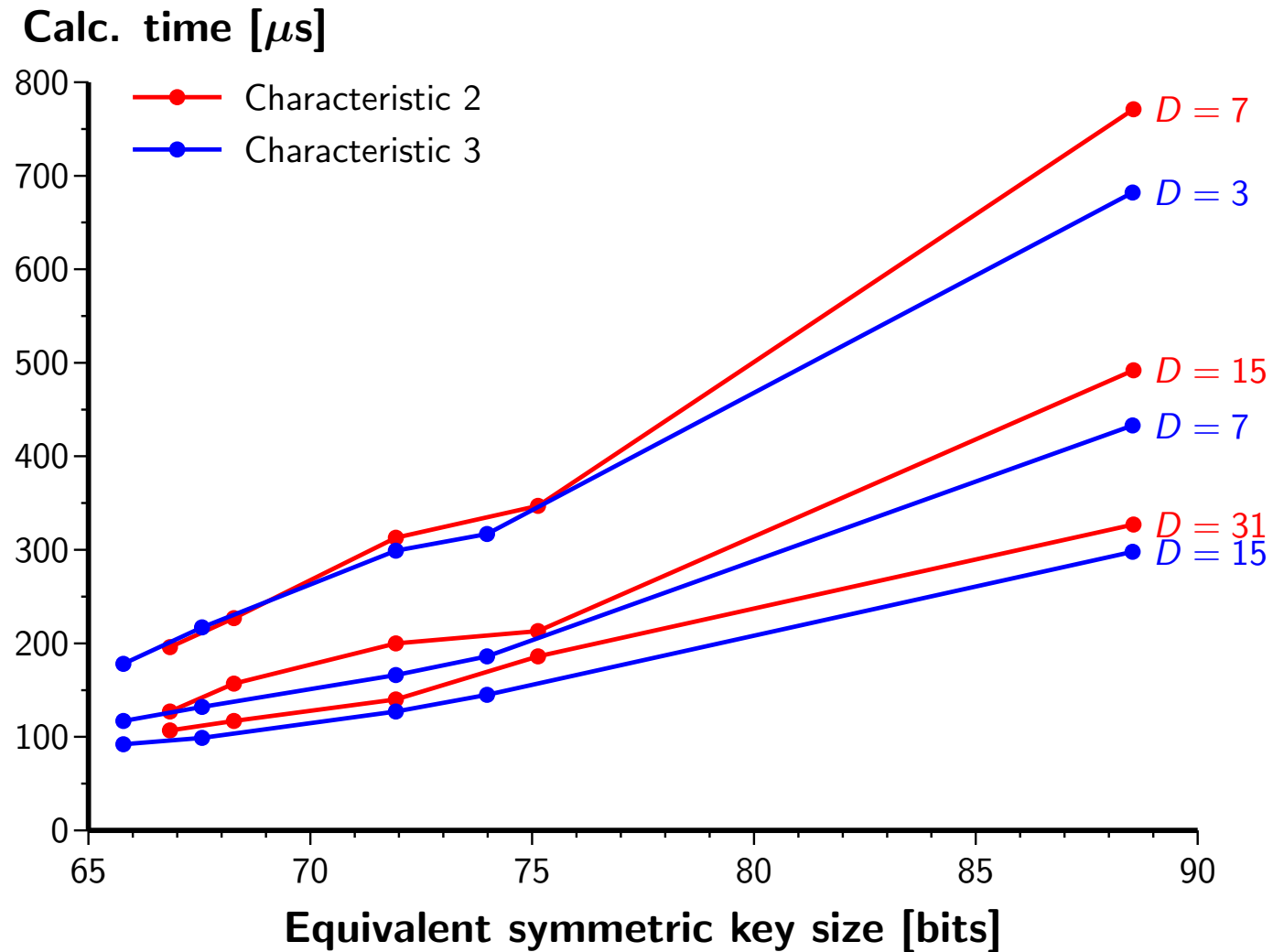
Calculation time (characteristic 2)



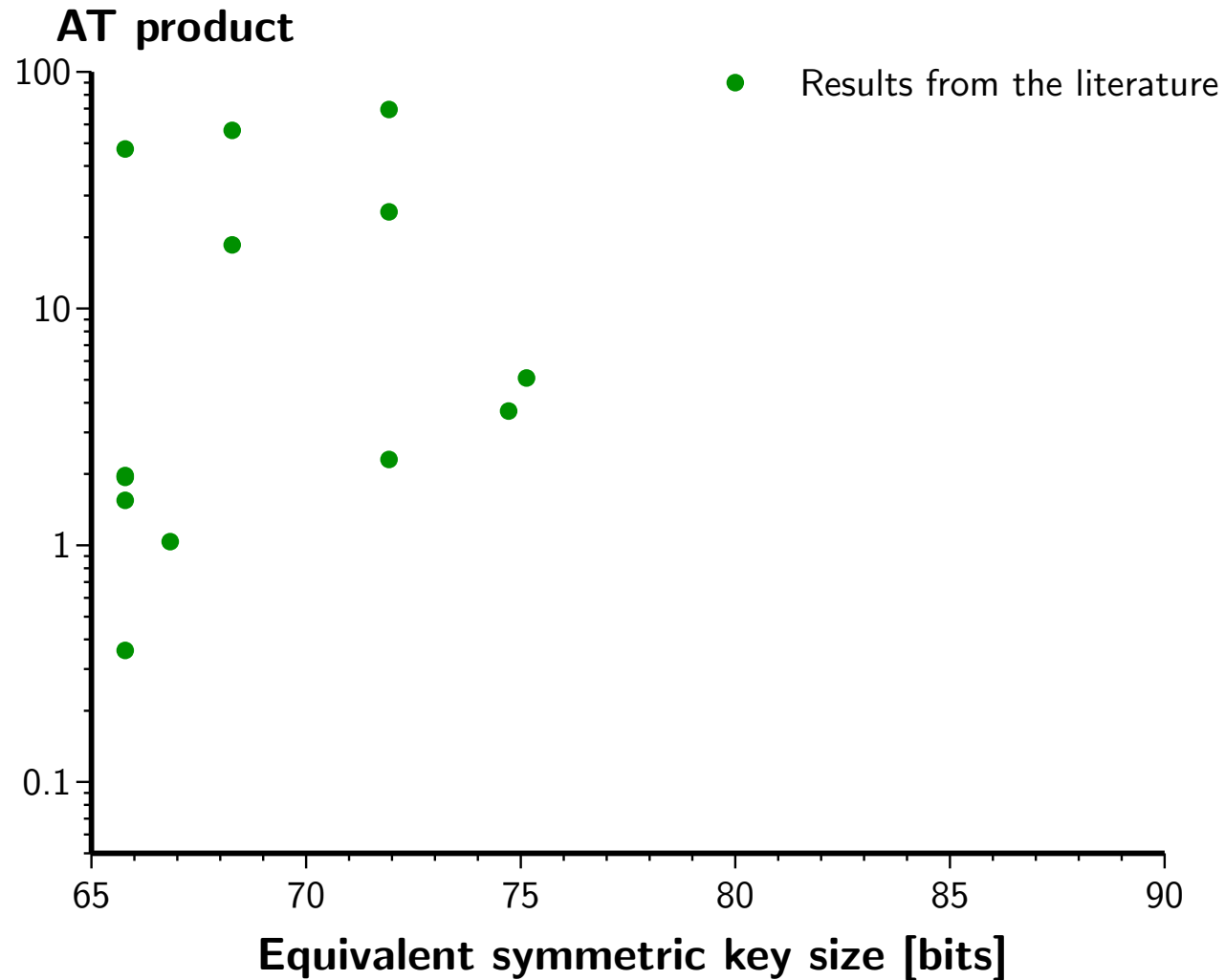
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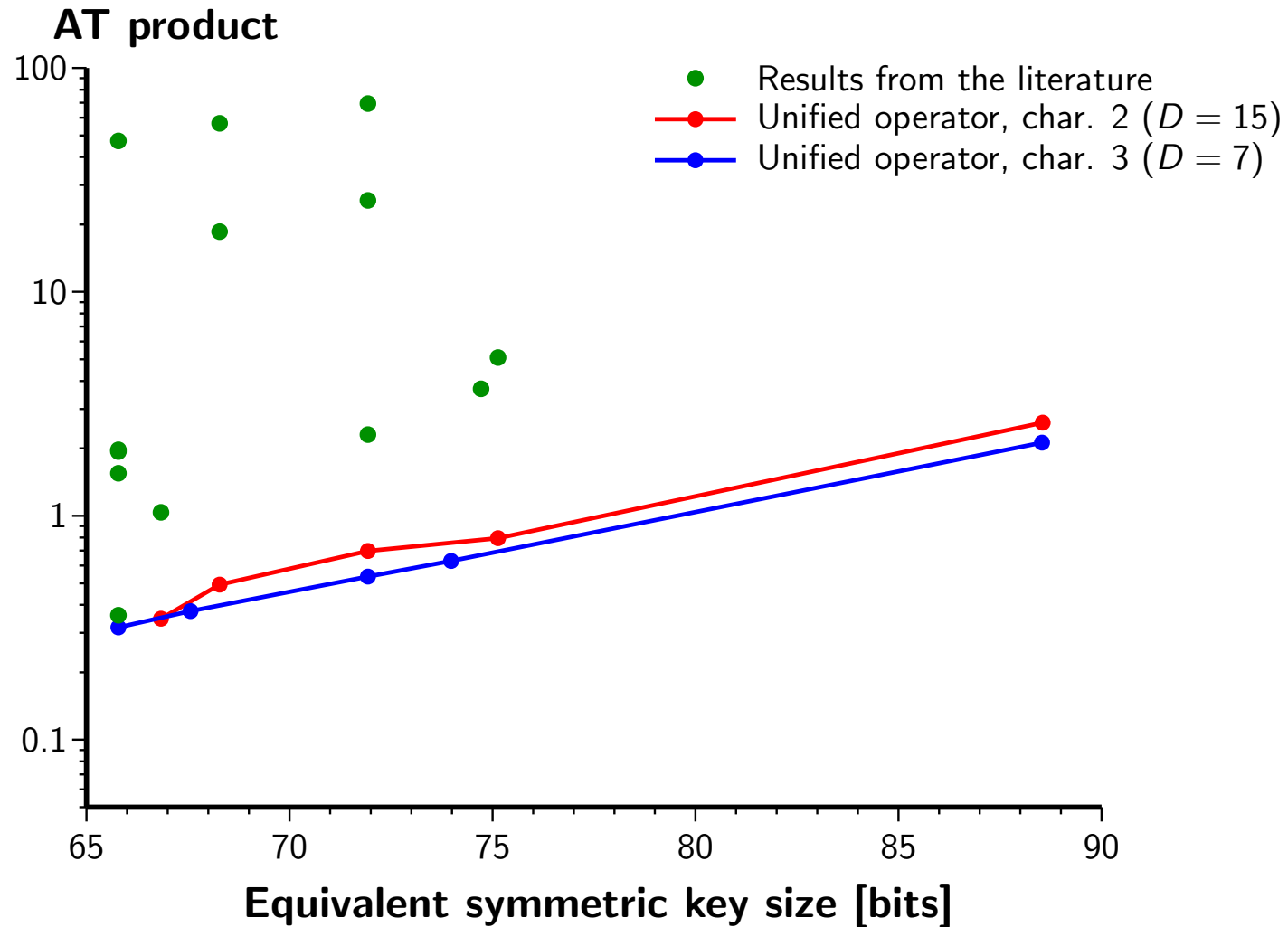
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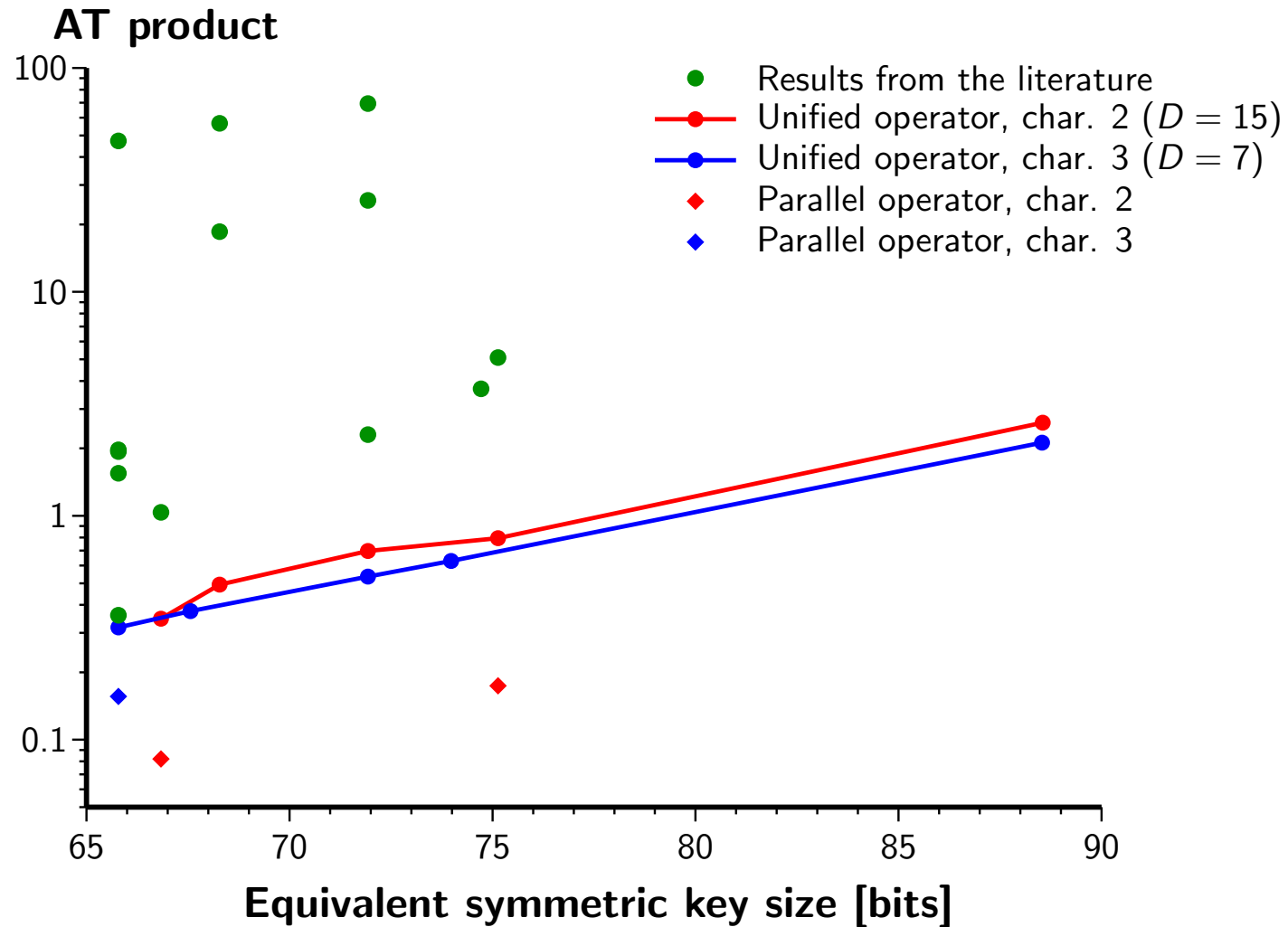
Comparison with published results



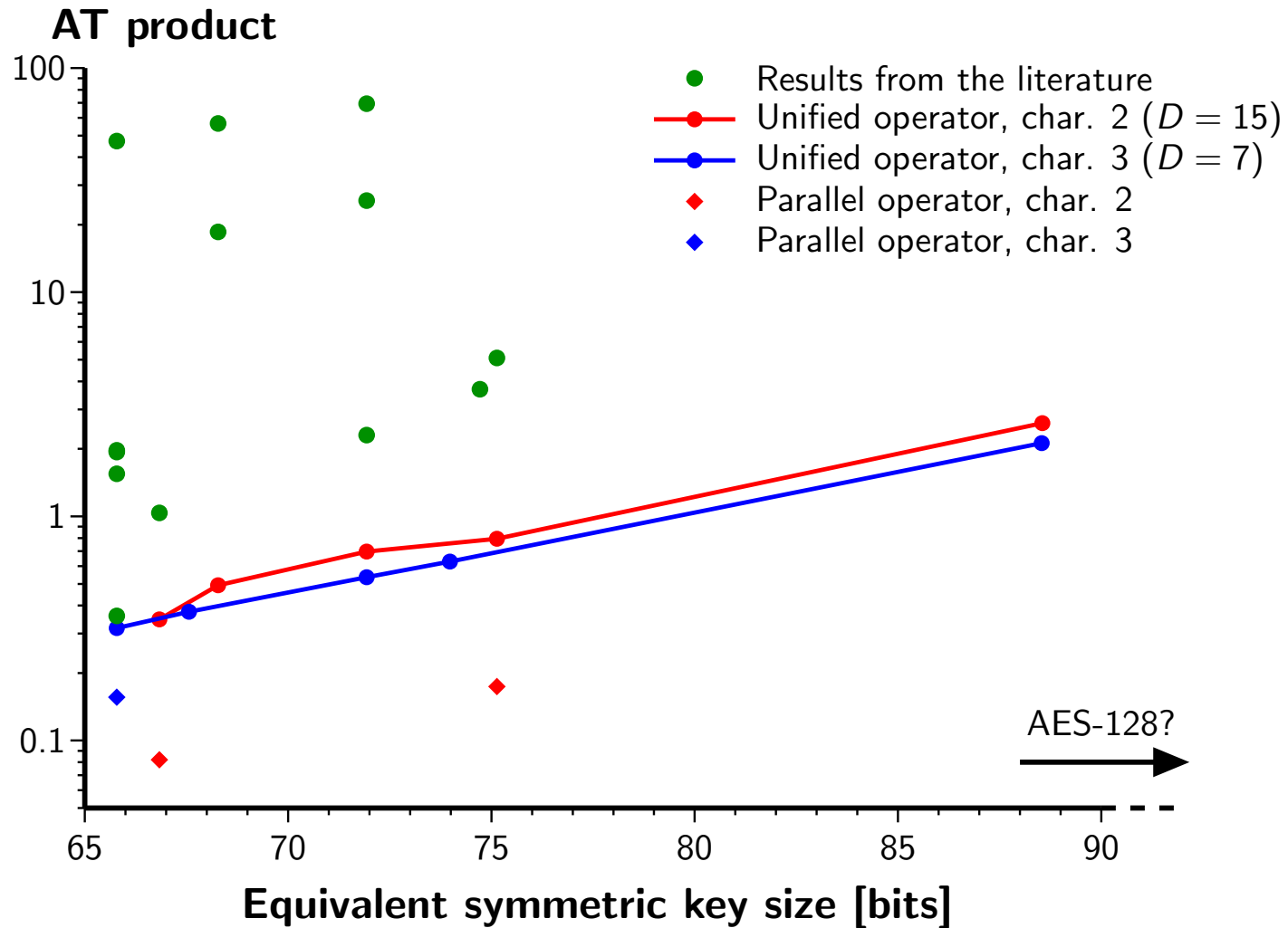
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 - **AES-128-equivalent security!**

With thanks to our sponsor



Thank you for your attention

Questions?